ASTR 101 — Final Exam Equation Sheet

Spring 2026 | Dr. Anna Rosen | Cumulative: Modules 1, 2, and 3

Light & Radiation

$$c = \lambda\nu$$

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = mc^2 \ \text{(mass–energy)}$$

$$\lambda(\text{nm}) = \frac{1240}{E(\text{eV})}$$

Thermal (Blackbody) Radiation

$$\lambda_{\text{peak}} = \frac{2.90\times10^6\ \text{nm}\cdot\text{K}}{T}$$

$$L = 4\pi R^2\sigma T^4$$

$$F_{\text{surface}} = \sigma T^4 \ \text{(per unit area)}$$

$$F = \frac{L}{4\pi d^2}$$

F = apparent brightness (flux at observer); d = distance

Atomic Spectra

$$E_n = -\frac{13.6\ \text{eV}}{n^2} \quad (n = 1, 2, 3, \ldots)$$

Bohr energy levels of hydrogen; photon energy of a transition is $|E_{\text{final}} - E_{\text{initial}}|$

Gravity & Orbits

$$F_g = \frac{GMm}{r^2} \qquad g = \frac{GM}{r^2}$$

$$a_c = \frac{v^2}{r}$$

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}} \qquad v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$$P^2 = a^3 \ \text{(yr, AU, around Sun)}$$

$$P^2 = \frac{4\pi^2}{GM}a^3 \ \text{(general)}$$

$$r_p = a(1-e) \qquad r_a = a(1+e)$$

Motion & Distance

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

$\Delta\lambda = \lambda_{\text{obs}} - \lambda_{\text{rest}}$; $v > 0$: recession

$$\text{distance} = \text{speed} \times \text{time}$$

Angular Geometry

$$\theta \approx \frac{D}{d} \ (\theta \text{ in radians})$$

D = physical size, d = distance; small angles

Telescope Performance

$$\theta_{\min} \approx 1.22\frac{\lambda}{D} \ \text{(resolution)}$$

$$\text{Light-gathering power} \propto D^2$$

Distance & Parallax

$$d(\text{pc}) = \frac{1}{p('')}$$

p in arcseconds; 1 pc $\approx$ 3.26 ly

Binary Stars

Total mass (Kepler III, solar units)

$$M_1 + M_2 = \frac{a^3}{P^2}$$

a in AU, P in yr, masses in $M_\odot$

Mass ratio (center of mass)

$$M_1 v_1 = M_2 v_2 \quad \frac{M_2}{M_1} = \frac{v_1}{v_2}$$

Main-Sequence Scaling

$$L \propto M^{3.5}$$

$$t_{\text{MS}} \propto \frac{M}{L} \propto M^{-2.5}$$

$$t_{\text{MS}}(1M_\odot) \approx 10^{10}\ \text{yr}$$

Compact-Object Limits

$$R_s = \frac{2GM}{c^2} \quad R_s \approx 3\ \text{km}\ (M/M_\odot)$$

$$R_{\text{WD}} \propto M_{\text{WD}}^{-1/3}$$

$$M_{\text{Ch}} \approx 1.4M_\odot \quad M_{\text{TOV}} \approx 2\text{–}3M_\odot$$

Galaxies — Mass from Motion

Rotation curve (enclosed mass)

$$M_{\text{enc}} = \frac{v^2 r}{G}$$

Flat rotation curves $v(r) \approx \text{const}$ imply $M_{\text{enc}(r)} \propto r$ → dark-matter halo

Eddington luminosity (radiation-pressure limit)

$$L_{\text{Edd}} \approx 1.3\times10^{31}\ \text{W}\ (M/M_\odot)$$

Minimum BH mass to power an observed luminosity L: $M \gtrsim (L/L_{\text{Edd}})M_\odot$

Cosmic Distance Ladder

Cepheid period–luminosity (Leavitt)

$$L \approx 335\, L_\odot (P/1\ \text{day})^{1.15}$$

Use measured period P (days) → luminosity, then inverse-square for d

Type Ia supernova standard candle

$$L_{\text{peak}} \approx 5\times10^9\, L_\odot$$

Phillips relation standardizes light curves; reach ~ 1000 Mpc

Hubble's Law & Cosmological Redshift

$$z = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0}$$

$$v \approx cz \quad (\text{low } z)$$

$$v = H_0\, d$$

d in Mpc, v in km/s, H_0 in km s^{-1} Mpc^{-1}

$$t_H = 1/H_0 \approx 14\ \text{Gyr}$$

$$d_H = c/H_0 \approx 4.3\ \text{Gpc}$$

Cosmic Microwave Background

$$T(z) = T_0(1+z)$$

CMB cools as space expands: $T_0 = 2.725$ K today, $T \approx 3000$ K at $z \approx 1100$ (recombination)

Qualitative Reference

Spectral Classification

Hot → Cool: O — B — A — F — G — K — M
O: $\gtrsim 30{,}000$ K G (Sun): ~ 5800 K M: ~ 3000 K

EM spectrum (low → high E): radio → microwave → IR → visible → UV → X-ray → gamma

Kirchhoff's Three Spectral Laws

- **Continuous:** hot, dense source → smooth blackbody.
- **Emission-line:** hot, low-density gas → bright lines at atomic transitions.
- **Absorption-line:** cool gas in front of continuous source → dark lines at the same wavelengths.

Cosmic Energy Budget

Ω-budget (today):
5% ordinary (baryonic) matter
27% dark matter
68% dark energy

Big Bang pillars: (1) Hubble expansion, (2) CMB blackbody, (3) primordial H/He/Li abundances.

Reference Tables

Constants, astrophysical values, math rules, and unit conversions

Physical Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^8 m/s
Gravitational const.	G	6.67×10^{-11} N m² kg⁻²
Stefan-Boltzmann	σ	5.67×10^{-8} W m⁻² K⁻⁴
Planck's constant	h	6.63×10^{-34} J s
Earth surface gravity	$g_\oplus$	9.8 m/s²
H ionization energy	$\lvert E_1 \rvert$	13.6 eV

Cosmological Reference Values

Quantity	Symbol	Value
Hubble constant	H_0	≈ 70 km s⁻¹ Mpc⁻¹
		(local: 73; CMB: 67)
Hubble time	$1/H_0$	≈ 14 Gyr
Age of universe	t_{univ}	13.8 Gyr
CMB temperature today	T_{CMB}	2.725 K
Recombination redshift	z_{rec}	~ 1100
Recombination temperature	T_{rec}	~ 3000 K
Sgr A* mass	$M_{\text{Sgr A}}$	$\sim 4 \times 10^6 M_\odot$
Type Ia peak	L_{peak}	$\sim 5 \times 10^9 L_\odot$

Astrophysical Reference Values

Quantity	Symbol	Value
Solar luminosity	$L_\odot$	3.8×10^{26} W
Solar radius	$R_\odot$	7.0×10^8 m
Solar mass	$M_\odot$	2.0×10^{30} kg
Solar surface temp.	$T_\odot$	5800 K
Solar peak wavelength	$\lambda_{\text{peak},\odot}$	≈ 500 nm
Solar core temp.	$T_{\text{core},\odot}$	$\sim 1.5 \times 10^7$ K
Solar MS lifetime	$t_{\text{MS},\odot}$	$\sim 10^{10}$ yr
Earth–Sun distance	1 AU	1.50×10^{11} m
Light-year	1 ly	9.46×10^{15} m
Parsec	1 pc	3.09×10^{16} m ≈ 3.26 ly
Megaparsec	1 Mpc	3.09×10^{22} m $= 10^6$ pc
Earth radius	$R_\oplus$	6.4×10^6 m
Earth mass	$M_\oplus$	6.0×10^{24} kg

Exponent & Scaling Rules

Rule	Example
$10^a \times 10^b = 10^{a+b}$	$10^3 \times 10^5 = 10^8$
$10^a / 10^b = 10^{a-b}$	$10^8 / 10^5 = 10^3$
$(10^a)^b = 10^{ab}$	$(10^3)^2 = 10^6$
$x^{1/2} = \sqrt{x}$	$4^{3/2} = 8$
$x^{-n} = 1/x^n$	$10^{-3} = 0.001$

Scaling / ratio method: if $A \propto B^n$, then

$$\frac{A_2}{A_1} = \left(\frac{B_2}{B_1}\right)^n$$

Example (Stefan-Boltzmann ratio):

$$\frac{L_2}{L_1} = \left(\frac{R_2}{R_1}\right)^2 \left(\frac{T_2}{T_1}\right)^4$$

Example (Hubble's law as ratio):

$$\frac{v_2}{v_1} = \frac{d_2}{d_1}$$

Unit Conversions

Category	Conversion
Angles	$1^\circ = 60' = 3600''$
	$1 \text{ rad} \approx 57.3^\circ \quad 2\pi \text{ rad} = 360^\circ$
Length	1 nm $= 10^{-9}$ m 1 km $= 10^3$ m
	1 pc ≈ 3.26 ly $= 3.09 \times 10^{16}$ m
	1 ly $\approx 9.46 \times 10^{15}$ m
	1 Mpc $= 10^6$ pc ≈ 3.26 Mly
Energy	1 eV $= 1.6 \times 10^{-19}$ J
	1 J $= 1$ kg m² s⁻²
Time	1 yr $\approx 3.15 \times 10^7$ s
	1 Gyr $= 10^9$ yr
Temperature	$T_K = T_C + 273$ (use K in Wien / Stefan-Boltzmann)