Name (Last, First): ____________________________

Astronomy 201: Midterm 1 Exam

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Instructions

- There are **two parts** to this exam, worth **56 points** total. Answer all questions in the space provided.
- Show your work in a clear and orderly fashion, including all unit conversions, for full credit on calculation-based questions. **Circle your final answers clearly.**
- Partial credit will be awarded where appropriate.
- A formula sheet is provided; identify and use the appropriate equations where necessary.

Part I: Multiple Choice Questions

Please show your work in the space provided if necessary and provide your answer on the line provided. *(20 points, 2 points each)*

1. Which of the following statements about Newton's Laws is true?

   (a) Newton's first law states that an object must always be in motion unless acted upon by an external force.
   (b) Newton's second law states that force is proportional to velocity.
   (c) Newton's third law states that forces always act in equal and opposite pairs.
   (d) Newton's laws state that the total energy of a system is always conserved.
   (e) Newton's laws do not apply in space because there is no gravity.

   Answer: ____________

   Answer: (c)

   Recall *Newton's Laws of Motion*:

   (a) *Law of Inertia:* An object at rest ($v = 0$) stays at rest, and an object in motion stays in motion with the same speed ($v =$ const.) and in the same direction unless acted upon by an unbalanced external force.
   (wrong choice **a**)
   (b) *Law of Acceleration:* The acceleration a of an object is directly proportional to the net force F acting on it and inversely proportional to its mass m: $F = ma$.
   (wrong choice **b**)
   (c) *Law of Action-Reaction:* For every action (force), there is an equal and opposite reaction (force). Forces always occur in pairs acting on different objects.
   (correct choice **c**)

2. Which of the following would increase the surface gravity of a planet?

   (a) Increasing the planet's radius while keeping its mass the same.
   (b) Decreasing the planet's mass while keeping its radius the same.
   (c) Increasing the planet's mass while keeping its radius the same.
   (d) Moving the planet farther from its star.
   (e) Changing the planet's rotation rate.

   Answer: ____________

   Answer: (c)

$$F_{\text{grav}} = \frac{GMm}{r^2} = mg \implies g = \frac{GM}{R^2} \implies g \propto M\,R^{-2}$$

   Surface gravity = surface acceleration due to gravity at $r = R$. Using **proportional reasoning**, for $g\uparrow$ requires $M\uparrow$ *or* $R\downarrow$. Hence, the options yield:

   (a) Increasing the planet's radius while keeping its mass the same.

$$R\uparrow, \quad M = \text{const.}; \quad \implies g\downarrow$$

   (b) Decreasing the planet's mass while keeping its radius the same.

$$R = \text{const.}, \quad M\downarrow; \quad \implies g\downarrow$$

   (c) Increasing the planet's mass while keeping its radius the same.

$$R = \text{const.}, \quad M\uparrow; \quad \implies g\uparrow$$

   (d) Moving the planet farther from its star. **(unrelated to surface gravity)**
   (e) Changing the planet's rotation rate. **(unrelated to surface gravity)**

3. A cloud of hot, low-density hydrogen gas is observed against a dark background. What type of spectrum would you expect to observe?

   (a) A continuous (blackbody) spectrum with no features.
   (b) A continuous spectrum with dark absorption lines.
   (c) Bright emission lines at specific wavelengths on a dark background.
   (d) A featureless flat spectrum at all wavelengths.

   Answer: ____________

   Answer: (c)

   Kirchhoff's Law 2: A hot, low-density (optically thin) gas emits light only at specific wavelengths corresponding to atomic transitions. Against a dark background, these appear as bright **emission lines** on a dark background.

- (a) describes Kirchhoff's Law 1 (hot, *dense* source), not a low-density gas.
- (b) describes Kirchhoff's Law 3 (cool gas in front of a hot, continuous source).
- (d) is not physical — all thermal sources have some spectral structure.

4. A star is observed to be very luminous ($L \gg L_\odot$) but has a cool surface temperature ($T_{\rm eff} \approx 3{,}500$ K). What can you conclude about this star?

   (a) It must have a very small radius.
   (b) It must have a very large radius.
   (c) It must be very close to Earth.
   (d) It must have a higher mass than the Sun.

   Answer: ____________

   Answer: (b)

   Stefan-Boltzmann Law:

   $$L = 4\pi\sigma R^2 T^4 \implies R^2 = \frac{L}{4\pi\sigma T^4}$$

   If L is very large and T is low, then R must be very large to compensate. This describes a **red giant or supergiant** — cool, luminous, and enormous.

   - (a) is the opposite: small radius with low T gives *low* luminosity (e.g., a red dwarf).
   - (c) is wrong: distance affects *apparent* brightness (flux), not *intrinsic* luminosity.
   - (d) is not necessarily true: red giants can have masses comparable to the Sun.

5. A spectral line normally at 600 nm is observed at 603 nm. What can be concluded about the motion of the source?

   (a) The source is moving toward Earth.
   (b) The source is moving away from Earth.
   (c) The source is stationary relative to Earth.
   (d) The source's temperature has increased.

   Answer: ____________

   Answer: (b)

   Doppler Shift:

   $$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

   $\Delta\lambda = 603\,\text{nm} - 600\,\text{nm} = +3\,\text{nm} > 0 \implies v_r > 0 \implies$ **moving away (redshifted)**

6. Which of the following expressions is dimensionally consistent (i.e., has the correct dimensions for the quantity on the left)?

   (a) Kinetic energy: $E = mv$ (where m is mass, v is velocity)
   (b) Gravitational PE: $U = GMm/r^2$ (where r is distance)
   (c) Orbital velocity: $v = \sqrt{GM/r}$ (where M is mass, r is distance)
   (d) Luminosity: $L = 4\pi R^2 T^4$ (where R is radius, T is temperature)

Answer: ___________

Answer: (c)

Check each option using $[G] = L^3 M^{-1} T^{-2}$ (length, mass, time):

- (a) mv has dimensions $[M \cdot L\,T^{-1}]$ = momentum, **not** energy $[M\,L^2\,T^{-2}]$. Missing a factor of v.
- (b) GMm/r^2 has dimensions:
$$\frac{[L^3\,M^{-1}\,T^{-2}]\,[M]\,[M]}{[L^2]} = [M\,L\,T^{-2}]$$
This is **force**, not energy $[M\,L^2\,T^{-2}]$. Should be GMm/r for PE.
- (c) $\sqrt{GM/r}$ has dimensions:
$$\sqrt{\frac{[L^3\,M^{-1}\,T^{-2}]\,[M]}{[L]}} = \sqrt{[L^2\,T^{-2}]} = [L\,T^{-1}]\ \checkmark$$
This is velocity. **Dimensionally correct.**
- (d) $R^2 T^4$ has dimensions $[L^2\,\Theta^4]$. Luminosity has dimensions $[M\,L^2\,T^{-3}]$ (power). Missing the constant σ, which carries the remaining dimensions.

7. In a vacuum, photons of higher energy

   (a) move faster than lower energy photons.
   (b) have higher frequencies and shorter wavelengths than lower energy photons.
   (c) have more mass than lower energy photons.
   (d) are not as likely to become redshifted as lower energy photons.
   (e) travel less distance between their source and observer than lower energy photons.

Answer: ___________

Answer: (b)

Photon Energy: $E = h\nu = \frac{hc}{\lambda}$, and $c = \lambda\nu$ = const. in vacuum.

Higher $E \implies$ higher $\nu \implies$ shorter λ. All photons travel at c in vacuum regardless of energy.

8. Two stars have measured parallaxes of 0.25 arcseconds and 0.05 arcseconds, respectively. How do their distances compare?

   (a) The star with the larger parallax is farther away.
   (b) The star with the smaller parallax is farther away.
   (c) Both stars are at the same distance.
   (d) Parallax cannot be used to determine distances.
   (e) The star with the smaller parallax is moving toward us.

   Answer: ____________

   Answer: (b)

   Parallax-Distance Relation:

$$d\ [\text{pc}] = \frac{1}{p\ [\text{arcsec}]} \implies d \propto p^{-1}$$

   Larger parallax $\implies$ *closer* (not farther). The star with $p = 0.25''$ is at $d = 4$ pc, while the star with $p = 0.05''$ is at $d = 20$ pc. The star with the **smaller** parallax is **farther** away.

9. A star has apparent magnitude $m = 8$ and absolute magnitude $M = 3$. Is this star closer to or farther from Earth than 10 pc?

   (a) Closer than 10 pc.
   (b) Farther than 10 pc.
   (c) Exactly at 10 pc.
   (d) Cannot be determined without knowing the star's luminosity.

   Answer: ____________

   Answer: (b)

   Distance Modulus:

$$m - M = 5\ \log_{10}\left(\frac{d}{10\ \text{pc}}\right)$$

   Here $m - M = 8 - 3 = +5 > 0$.

   Since $m > M$, the star appears *dimmer* than it would at the standard distance of 10 pc, which means it must be **farther** than 10 pc. (In fact, $m - M = 5$ gives $d = 100$ pc.)

   - (a) is backwards: $m < M$ (star appears brighter than at 10 pc) would mean closer.
   - (c) would require $m = M$ (distance modulus = 0).
   - (d) is wrong: the distance modulus relates m, M, and d directly — no additional luminosity information needed.

10. In a double-lined spectroscopic binary, Star 1 has a radial velocity amplitude $K_1 = 90$ km/s and Star 2 has $K_2 = 30$ km/s. Which star is more massive?

(a) Star 1 is more massive.

(b) Star 2 is more massive.

(c) Both stars have equal mass.

(d) Mass cannot be determined from velocity amplitudes alone.

Answer: ____________

Answer: (b)

Center-of-Mass / Velocity-Mass Relation:

$$\frac{K_1}{K_2} = \frac{M_2}{M_1} \implies \frac{M_2}{M_1} = \frac{90 \text{ km/s}}{30 \text{ km/s}} = 3$$

Star 2 is 3$\times$ more massive than Star 1. The more massive star orbits closer to the center of mass and therefore moves **slower** (smaller K).

- (a) reverses the relation: the faster-moving star is the *less* massive one.
- (c) would require $K_1 = K_2$.
- (d) is wrong: in a double-lined (SB2) binary, the velocity ratio *directly* gives the mass ratio.

Part II: Problems

Tip: *Solve algebraically before plugging in numbers! This helps minimize errors and ensures that if you make a mistake, you can still earn credit for correctly deriving later expressions.*

1) Two stars, Star A and Star B, have the following properties:

- **Star A is three times as far away as Star B:** $d_A = 3d_B$.
- **Both stars have the same observed flux:** $F_A = F_B$.
- **The measured peak wavelength of Star B is one-third that of Star A:** $\lambda_B = \frac{1}{3}\lambda_A$.

Using these relationships, express the following properties of **Star A in terms of Star B**:

(a) Express the luminosity of Star A (L_A) in terms of the luminosity of Star B (L_B). *(4 pts)*

Solution:

Observed Flux (*Apparent Brightness*):

$$F = \frac{L}{4\pi d^2} \implies L = 4\pi d^2 F \implies L \propto d^2 F$$

Given: $F_A = F_B$ & $d_A = 3d_B$

Using the ratio method:

$$\frac{L_A}{L_B} = \frac{d_A^2 \cancel{F_A}}{d_B^2 \cancel{F_B}} = \frac{(3d_B)^2}{d_B^2} = \frac{9\cancel{d_B^2}}{\cancel{d_B^2}} = 9$$

$$\implies \boxed{L_A = 9L_B}$$

(b) Express the effective temperature of Star A (T_A) in terms of the effective temperature of Star B (T_B). *(4 pts)*

Solution:
Given: $\lambda_B = \frac{1}{3}\lambda_A$.

$$\textbf{Wien's Law:} \quad \lambda_{\text{peak}} = \frac{b}{T_{\text{eff}}} \implies \lambda_{\text{peak}} \propto T_{\text{eff}}^{-1}$$

Using the ratio method:

$$\frac{T_A}{T_B} = \frac{\lambda_{\text{peak,B}}}{\lambda_{\text{peak,A}}} = \frac{\frac{1}{3}\cancel{\lambda_A}}{\cancel{\lambda_A}} = \frac{1}{3}$$

$$\implies \boxed{T_A = \frac{1}{3}T_B}$$

Sanity check: Star A has a longer peak wavelength, so it should be cooler. $T_A < T_B$ ✓

(c) Express the radius of Star A (R_A) in terms of the radius of Star B (R_B). *(4 pts)*

Solution:

$$\textbf{Stefan-Boltzmann Law:} \quad L = 4\pi\sigma R^2 T^4 \implies L \propto R^2 T^4$$

Using the ratio method:

$$\frac{L_A}{L_B} = \left(\frac{R_A}{R_B}\right)^2 \left(\frac{T_A}{T_B}\right)^4$$

Solving for R_A/R_B:

$$\left(\frac{R_A}{R_B}\right)^2 = \frac{L_A}{L_B}\left(\frac{T_A}{T_B}\right)^{-4} \implies \frac{R_A}{R_B} = \left(\frac{L_A}{L_B}\right)^{1/2}\left(\frac{T_A}{T_B}\right)^{-2}$$

Plugging in our previous answers ($L_A = 9L_B$, $T_A = \frac{1}{3}T_B$):

$$\frac{R_A}{R_B} = (9)^{1/2}\left(\frac{1}{3}\right)^{-2} = 3 \times 9 = 27$$

$$\implies \boxed{R_A = 27\, R_B}$$

Sanity check: Star A is cooler but equally bright (same flux) and farther away, so it must be more luminous (9×). To be more luminous despite being cooler requires a much larger radius. ✓

2) Consider two exoplanets, **Planet A** and **Planet B**, orbiting a star with mass $M_\star = 2\ M_\odot$.

The planets have the following orbital properties:

- Planet B orbits at twice the semi-major axis of Planet A: $a_B = 2\,a_A$.
- Both planets have circular orbits.
- Planet A completes one full orbit in 1 Earth year.
- Both planets have the same mass: $M_A = M_B$.

Using this information, answer the following questions:

(a) How do the orbital periods of the two planets compare? Express P_B in terms of P_A. *(4 pts)*

Solution:

Kepler's 3rd Law:

$$P^2 = \frac{4\pi^2}{GM_\star} a^3 \implies P \propto a^{3/2}$$

(The $M_\star$ factor cancels since both planets orbit the same star.)

Using the ratio method:

$$\frac{P_B}{P_A} = \left(\frac{a_B}{a_A}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2} \approx 2.83$$

$$\implies \boxed{P_B = 2\sqrt{2}\,P_A \approx 2.83\ \text{yr}}$$

Sanity check: Planet B is farther out, so it should have a longer period. ✓

(b) How do the orbital velocities of Planet A and Planet B compare? Express v_A in terms of v_B. *(4 pts)*

Solution:

Orbital Velocity:

$$v_{\text{orb}} = \sqrt{\frac{GM_\star}{r}} \implies v_{\text{orb}} \propto r^{-1/2}$$

(Again, $M_\star$ cancels in the ratio since both orbit the same star.)

Using the ratio method with $r = a$:

$$\frac{v_{\text{orb,A}}}{v_{\text{orb,B}}} = \left(\frac{a_B}{a_A}\right)^{1/2} = (2)^{1/2} = \sqrt{2}$$

$$\implies \boxed{v_{\text{orb,A}} = \sqrt{2}\,v_{\text{orb,B}}}$$

Sanity check: The closer planet orbits faster. ✓

(c) How does the gravitational force exerted by the star on each planet compare? Express $F_{g,A}$ in terms of $F_{g,B}$. *(4 pts)*

Solution:

Gravitational Force:

$$F_g = \frac{GM_\star m}{r^2} \implies F_g \propto m\, r^{-2}$$

Since $M_A = M_B$ (given) and both orbit the same star ($M_\star$ cancels):

$$\frac{F_{g,A}}{F_{g,B}} = \frac{\cancel{M_\star}\,\cancel{M_A}\, a_A^{-2}}{\cancel{M_\star}\,\cancel{M_B}\, a_B^{-2}} = \left(\frac{a_B}{a_A}\right)^2 = 2^2 = 4$$

$$\implies \boxed{F_{g,A} = 4\, F_{g,B}}$$

Sanity check: The closer planet experiences a stronger gravitational force (inverse-square law). ✓

3) The H-alpha (Hα) emission line is a spectral line of hydrogen with a rest wavelength of $\lambda_0 = 656.3$ nm. Astronomers observe a binary star system and measure the Hα spectral lines of both stars. The first star's Hα line appears at 658.2 nm, while the second star's Hα line appears at 654.5 nm.

(a) Determine the motion and radial (line-of-sight) velocity of each star. (1) For each star, state whether it is moving **toward** or **away** from Earth and whether its spectral line is **redshifted** or **blueshifted**. (2) Then, calculate the **radial velocity** of each star relative to Earth. Express your answers both as a fraction of the speed of light, c, and in km/s. *(4 pts)*

Solution:

Doppler Shift:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{v_r}{c} \implies v_r = \frac{\Delta\lambda}{\lambda_0} c$$

- **Star 1:** $\lambda_{\text{obs}} = 658.2$ nm

$$\Delta\lambda = 658.2 \text{ nm} - 656.3 \text{ nm} = +1.9 \text{ nm} \implies v_r > 0$$

$$\boxed{\textbf{Star 1: moving away from Earth; redshifted}}$$

$$v_{r,1} = \frac{1.9 \cancel{\text{nm}}}{656.3 \cancel{\text{nm}}} c = 0.0029\, c = 0.0029 \times 3 \times 10^5 \text{ km/s} \approx 868 \text{ km/s}$$

$$\implies \boxed{v_{r,1} \approx 0.0029\, c \approx 868 \text{ km/s}}$$

- **Star 2:** $\lambda_{\text{obs}} = 654.5$ nm

$$\Delta\lambda = 654.5 \text{ nm} - 656.3 \text{ nm} = -1.8 \text{ nm} \implies v_r < 0$$

$$\boxed{\textbf{Star 2: moving toward Earth; blueshifted}}$$

$$v_{r,2} = \frac{-1.8 \cancel{\text{nm}}}{656.3 \cancel{\text{nm}}} c = -0.0027\, c = -0.0027 \times 3 \times 10^5 \text{ km/s} \approx -823 \text{ km/s}$$

$$\implies \boxed{v_{r,2} \approx -0.0027\, c \approx -823 \text{ km/s}}$$

Sanity check: Both velocities are $\ll c$, so the non-relativistic Doppler formula is valid. ✓

(b) If the same stars were observed using the Hβ spectral line ($\lambda_0 = 486.1$ nm), would the measured wavelength shifts $\Delta\lambda$ be the same as those for Hα? Explain why or why not. *(4 pts)*

Solution:

No, the wavelength shifts would be **different** (smaller).

From the Doppler formula:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c} \implies \Delta\lambda = \lambda_0 \cdot \frac{v_r}{c}$$

The *fractional* shift $\Delta\lambda/\lambda_0 = v_r/c$ is the same for both lines (since the stars' velocities haven't changed), but the *absolute* shift $\Delta\lambda$ is proportional to λ_0. Since $\lambda_{\beta,0} = 486.1$ nm $<$ $\lambda_{\alpha,0} = 656.3$ nm, the absolute wavelength shift for Hβ would be smaller.

(c) If the binary system were observed **face-on instead of edge-on**, how would this affect the measured Doppler shifts? Would the shifts be **larger, smaller, or unchanged**? Explain your reasoning. *(4 pts)*
Tip: *A diagram may help.*

Solution:

The shifts would be **smaller** — approaching zero.

The Doppler effect only measures the *radial* (line-of-sight) component of velocity. In an edge-on view ($i = 90°$), the full orbital velocity projects along the line of sight, giving the maximum Doppler shift. In a face-on view ($i = 0°$), the orbital motion is entirely in the plane of the sky (transverse), so there is *no* line-of-sight component and $\Delta\lambda \to 0$.

Mathematically, the observed radial velocity is $v_r = v_{\text{orb}} \sin i$, so as $i \to 0°$, $\sin i \to 0$ and $v_r \to 0$.

Astronomy 201 Formula Sheet

Midterm 1 · Modules 1 & 2 · CGS Units

Electromagnetic Radiation

Speed of Light

$$c = \lambda \nu$$

λ [cm] = wavelength ν [Hz = s^{-1}] = frequency

Photon Energy

$$E = h\nu = \frac{hc}{\lambda}$$

Planck Function (Blackbody Spectrum)

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/k_B T} - 1} \ \ [\text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1}\ \text{sr}^{-1}]$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda k_B T} - 1} \ \ [\text{erg s}^{-1}\ \text{cm}^{-3}\ \text{sr}^{-1}]$$

Wien's Displacement Law

$$\lambda_{\text{peak}} = \frac{2.898 \times 10^6\ \text{nm}\cdot\text{K}}{T}$$

Stefan-Boltzmann Law

$$F_\star = \sigma T_{\text{eff}}^4 \ \ [\text{erg s}^{-1}\ \text{cm}^{-2}]$$

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \ \ [\text{erg s}^{-1}]$$

Planetary Equilibrium Temperature

$$T_{\text{eq}} = \left(\frac{L_\star (1 - A)}{16\pi \sigma d^2} \right)^{1/4}$$

A = albedo

Brightness & Magnitudes

Apparent Brightness (Observed Flux)

$$F = \frac{L}{4\pi d^2} \ \ [\text{erg s}^{-1}\ \text{cm}^{-2}]$$

Magnitude System

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

Distance Modulus

$$m - M = 5 \log_{10}\left(\frac{d}{10\ \text{pc}}\right)$$

Absolute Magnitude from Luminosity

$$M = 4.83 - 2.5 \log_{10}\left(\frac{L}{L_\odot}\right)$$

where $M_{V,\odot} = 4.83$.

Angular Size & Parallax

Small-Angle Approximation

$$\alpha = \frac{s}{d} \ \ [\text{rad}]$$

s = physical size d = distance

1 rad = 206,265 arcsec

Trigonometric Parallax

$$d = \frac{b}{p}\ ; \quad d\ [\text{pc}],\ p\ [\text{arcsec}],\ b\ [\text{AU}]$$

Kirchhoff's Laws of Spectroscopy

1. Hot, dense source → **continuous spectrum**
2. Hot, low-density gas → **emission lines**
3. Cool gas before hot source → **absorption lines**

Gravity

Newton's Second Law

$$F = m\, a \ \ [\text{dyne} = \text{g cm s}^{-2}]$$

Gravitational Force

$$F_g = \frac{G\, m_1\, m_2}{r^2}$$

r = center-to-center separation

Surface Gravity

$$g = \frac{GM}{R^2} \ \ [\text{cm s}^{-2}]$$

Orbital Speed ($m \ll M$, circular)

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}} \ \ [\text{cm s}^{-1}]$$

Escape Speed

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \ \ [\text{cm s}^{-1}]$$

Center of Mass

$$M_1\, a_1 = M_2\, a_2, \qquad a = a_1 + a_2$$

a_1, a_2 = distances of M_1, M_2 from center of mass

Energy & Virial Theorem

Kinetic Energy

$$E_K = \frac{1}{2} m\, v^2 \ \ [\text{erg}]$$

Gravitational Potential Energy

$$U(r) = -\frac{GMm}{r} \qquad (U \to 0 \text{ as } r \to \infty)$$

Total Energy

$$E = E_K + U$$

$E < 0$: bound $E = 0$: escape $E > 0$: unbound

Virial Theorem (Bound Equilibrium)

$$2\, E_K + U = 0$$

Kepler's Laws & Binary Orbits

Kepler's Third Law (Newton's Form)

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)}\, a^3$$

P [s] = orbital period a [cm] = semi-major axis

Convenient Solar-Unit Form

$$\frac{M_1 + M_2}{M_\odot} = \frac{(a/\text{AU})^3}{(P/\text{yr})^2}$$

Spectroscopic Binary Velocities

$$\frac{K_1}{K_2} = \frac{M_2}{M_1}$$

$K_1,\ K_2$ = velocity amplitudes

Mass-Luminosity Relation (Main Sequence)

$$L \propto M^{3.5}$$

Main-Sequence Lifetime

$$t_{\mathrm{MS}} \propto \frac{M}{L} \propto M^{-2.5}$$

Bound Orbit Energy

$$E = -\frac{GMm}{2a}$$

a = semi-major axis

Inferring Motion

Doppler Effect (non-relativistic, $v_r \ll c$)

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\mathrm{obs}} - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

λ_0 = rest wavelength λ_{obs} = observed wavelength

Telescopes

Collecting Area

$$A = \frac{\pi}{4} D^2$$

D = aperture diameter

Angular Resolution (Diffraction Limit)

$$\alpha_\lambda = 1.22\,\frac{\lambda}{D}\ \text{[rad]}$$

Spectral Resolution

$$R = \frac{\lambda}{\Delta\lambda}$$

Mathematical Formulae

Powers

$$y^a \cdot y^b = y^{a+b} \qquad y^a/y^b = y^{a-b}$$
$$(y^a)^b = y^{ab} \qquad y^{1/n} = \sqrt[n]{y}$$
$$y^{-a} = 1/y^a \qquad y^0 = 1$$

Logarithms

$$\log(a \times b) = \log a + \log b$$
$$\log(a/b) = \log a - \log b$$
$$\log(a^b) = b\,\log a$$

$x = 10^a \Leftrightarrow \log_{10} x = a$ $y = e^b \Leftrightarrow \ln y = b$

Sphere Geometry

$V = \frac{4}{3}\pi r^3$ (volume) $A_s = 4\pi r^2$ (surface area)

$A_p = \pi r^2$ (cross-section) $C = 2\pi r$ (circumference)

Physical Constants (CGS)

Speed of light	$c = 3.0 \times 10^{10}$ cm s^{-1}
	$= 3.0 \times 10^5$ km s^{-1}
Gravitational const.	$G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}
Boltzmann const.	$k_B = 1.38 \times 10^{-16}$ erg K^{-1}
Stefan-Boltzmann	$\sigma = 5.67 \times 10^{-5}$
	erg cm^{-2} s^{-1} K^{-4}
Planck's const.	$h = 6.63 \times 10^{-27}$ erg s
Proton mass	$m_p \approx 1.67 \times 10^{-24}$ g
Electron mass	$m_e = 9.11 \times 10^{-28}$ g
1 eV	$= 1.60 \times 10^{-12}$ erg
hc	$= 1{,}240$ eV·nm

Astronomical Values & Conversions

Solar Values

$M_\odot$	2.0×10^{33} g
$R_\odot$	7.0×10^{10} cm
$T_{\mathrm{eff},\odot}$	5800 K
$L_\odot$	3.8×10^{33} erg s^{-1}

Planetary Values

M_{Jup}	1.9×10^{30} g
R_{Jup}	7.0×10^9 cm
$M_\oplus$	6.0×10^{27} g
$R_\oplus$	6.4×10^8 cm

Distances

1 pc	3.086×10^{18} cm = 3.26 ly
1 ly	9.46×10^{17} cm
1 AU	1.496×10^{13} cm
1 yr	3.156×10^7 s

Angular Measure

1°	60 arcmin
$1'$	60 arcsec
1 rad	206,265 arcsec

CGS base units	cm, g, s
1 dyne (force)	g cm s^{-2}
1 erg (energy)	g cm^2 s^{-2}

n = 10^{-9}	k = 10^3	M = 10^6
$\mu = 10^{-6}$	m = 10^{-3}	G = 10^9