ASTR 201 — Midterm 2 Exam Solutions

Astronomy for Science Majors — Spring 2026

San Diego State University

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Instructions:

- There are **two parts** to this exam, worth **50 points** total. Answer all questions in the space provided.
- Show your work in a clear and orderly fashion, including all unit conversions, for full credit on calculation-based questions. **Circle your final answers clearly.**
- Partial credit will be awarded where appropriate.
- A formula sheet is provided; identify and use the appropriate equations where necessary. CGS units are used throughout.
- Use **proportional reasoning, ratio methods**, and scaling arguments wherever possible — full derivations are not required unless specifically requested.

Part I: Multiple Choice Questions

Please show your work in the space provided if necessary and provide your answer on the line provided. *(14 points, 2 points each)*

1. For a main-sequence star, which of the following is the correct ordering of the three fundamental stellar timescales from **shortest to longest**?

(a) $\tau_{\rm nuc} < \tau_{\rm KH} < \tau_{\rm dyn}$
(b) $\tau_{\rm KH} < \tau_{\rm dyn} < \tau_{\rm nuc}$
(c) $\tau_{\rm dyn} < \tau_{\rm KH} < \tau_{\rm nuc}$
(d) $\tau_{\rm dyn} < \tau_{\rm nuc} < \tau_{\rm KH}$
(e) All three are comparable on the main sequence.

Answer: ____________

Answer: (c)

The **dynamical timescale** $\tau_{\rm dyn} \sim 1/\sqrt{G\bar{\rho}} \approx 30\ \rm min$ (Sun) is set by free-fall under gravity alone — the shortest. The **Kelvin-Helmholtz timescale** $\tau_{\rm KH} \sim GM^2/(RL) \approx 30\ \rm Myr$ (Sun) is the time to radiate away the star's gravitational binding energy. The **nuclear timescale** $\tau_{\rm nuc} \sim \varepsilon f M c^2/L \approx 10\ \rm Gyr$ (Sun) is set by available fusion fuel.

Hierarchy: $\tau_{\rm dyn} \ll \tau_{\rm KH} \ll \tau_{\rm nuc}$. The large separation is why stars can be treated as in hydrostatic equilibrium on nuclear-burning timescales.

2. A 10 $M_{\rm sun}$ main-sequence star has a lifetime approximately how long compared to the Sun's?

(a) $10 \times$ longer (100 Gyr).
(b) $\approx 3 \times$ shorter (3 Gyr).
(c) $\approx 300 \times$ shorter (30 Myr).
(d) $\approx 3 \times 10^5 \times$ shorter (30 kyr).
(e) Its lifetime is independent of mass.

Answer: ____________

Answer: (c)

Main-sequence lifetime scaling: $\tau_{\rm MS} \propto M/L \propto M/M^{3.5} = M^{-2.5}$.

$$\tau_{\rm MS}\frac{10M_{\rm sun}}{\tau_{\rm MS}}(M_{\rm sun}) = (10)^{-2.5} \approx \frac{1}{316} \approx 3 \times 10^{-3}$$

With $\tau_{\rm MS,sun} \approx 10\ \rm Gyr \implies \tau_{\rm MS}(10M_{\rm sun}) \approx 30\ \rm Myr$. More massive stars burn fuel vastly faster because L rises faster than M.

3. If mass is gradually added to a main-sequence star (assume R stays roughly constant), how does the core respond to maintain hydrostatic equilibrium?

(a) Core expands; T_c decreases; P_c decreases.
(b) Core contracts; T_c decreases; P_c increases.
(c) Core expands; T_c increases; P_c decreases.
(d) Core contracts; T_c increases; P_c increases.
(e) Core properties remain unchanged.

Answer: ____________

Answer: (d)

Holding the **stellar** radius R fixed while adding mass, the homologous one-zone scalings immediately give

$$P_c \sim GM^2/R^4 \quad \Longrightarrow \quad P_c \uparrow \text{when } M \uparrow$$

$$T_c \sim \mu G M m_p/(k_B R) \quad \Longrightarrow \quad T_c \uparrow \text{when } M \uparrow$$

$$\rho_c \sim M/R^3 \quad \Longrightarrow \quad \rho_c \uparrow \text{when } M \uparrow$$

Physically, the **core itself contracts** (a sub-region of the star shrinks while the envelope adjusts to keep R fixed): more mass means more weight to support, so ρ_c must rise — and gravitational compression of the core heats it ($T_c \uparrow$) by the virial theorem.

4. Quantum tunneling allows protons to fuse in the Sun's core, even though the classical thermal energy ($k_B T_c \approx 1$ keV) is far below the Coulomb barrier ($E_C \approx 1$ MeV). Why is this possible?

(a) Protons have zero size and so do not feel the Coulomb barrier.
(b) Gravity is strong enough at the core to push protons together.
(c) Protons behave as quantum waves, giving a nonzero probability of penetrating the Coulomb barrier.
(d) Magnetic fields in the core channel protons toward one another.
(e) The CNO cycle lowers the temperature threshold for fusion.

Answer: ____________

Answer: (c)

At $T_c \approx 1.5 \times 10^7$ K, $k_B T \approx 1$ keV — three orders of magnitude below the Coulomb barrier (~ 1 MeV). Classically, protons never fuse. But each proton has a de Broglie wavelength $\lambda_{\text{dB}} = h/p$ comparable to the nuclear-approach distance, so the proton has a nonzero **amplitude** to be found on the other side of the barrier. The Gamow factor $\Gamma \propto e^{-2G}$ quantifies this tunneling probability.

5. Fusion of elements heavier than iron (^{56}Fe) does not release energy in stellar cores. Why?

(a) The Coulomb barrier becomes infinitely large for $Z > 26$.
(b) Nuclei heavier than iron have lower binding energy per nucleon, so fusing them absorbs rather than releases energy.
(c) All available fuel is locked up in iron.
(d) Neutrino losses prevent further reactions.
(e) Heavy nuclei spontaneously fission before they can fuse.

Answer: ____________

Answer: (b)

The binding energy per nucleon peaks near ^{56}Fe. For $A < 56$, fusion **releases** energy (exothermic, $\Delta m > 0$). For $A > 56$, fusion **absorbs** energy — so it cannot be a stellar energy source. Iron is the "ash" of stellar burning.

6. Light travels from the Sun's core to its surface in roughly 100,000 years, even though light travels at speed c and $R_{\rm sun} \approx 7 \times 10^{10}$ cm (a light-crossing time of ~ 2 s). What is the physical reason?

(a) Photons slow down when passing through dense plasma.
(b) The photons lose energy and are recreated at each absorption.
(c) Photons undergo a random walk, being absorbed and re-emitted $\sim 10^{24}$ times, so energy diffuses outward rather than streaming.
(d) Gravitational time dilation slows photons near the core.
(e) Convection carries photons slowly from the core.

Answer: ____________

Answer: (c)

Photon mean free path in the solar core is $\ell = 1/(\kappa\rho) \approx 0.02$ cm. To traverse $R_{\rm sun} = 7 \times 10^{10}$ cm by random walk requires $N \sim (R/\ell)^2 \approx 10^{24}$ steps, giving a diffusion time $\sim R^2/(\ell c) \sim 10^5$ yr. Photons travel at c between scatterings, but the **energy** diffuses slowly because the walk is random.

7. Which is the correct evolutionary sequence for a $1M_{\text{sun}}$ star **after** it leaves the main sequence?

(a) Main-sequence turnoff $\rightarrow$ red giant branch $\rightarrow$ helium flash $\rightarrow$ horizontal branch $\rightarrow$ AGB $\rightarrow$ planetary nebula $\rightarrow$ white dwarf.
(b) Main-sequence turnoff $\rightarrow$ white dwarf $\rightarrow$ red giant $\rightarrow$ planetary nebula.
(c) Red giant $\rightarrow$ main sequence $\rightarrow$ horizontal branch $\rightarrow$ white dwarf.
(d) Main-sequence turnoff $\rightarrow$ AGB $\rightarrow$ red giant $\rightarrow$ helium flash $\rightarrow$ white dwarf.
(e) Main-sequence turnoff $\rightarrow$ supernova $\rightarrow$ neutron star.

Answer: ____________

Answer: (a)

Low-mass post-MS evolution: core H is exhausted, the core contracts and H-shell burning begins, the envelope expands up the **red giant branch**. The He core becomes degenerate; when T_c reaches 10^8 K, **helium flash** ignites triple-α briefly. The star settles on the **horizontal branch** (core He burning). After core He exhausts, H + He shell burning drives the **asymptotic giant branch**; intense mass loss ejects the envelope as a **planetary nebula**, leaving a hot C/O **white dwarf** to cool. Option (e) is for massive stars ($M \gtrsim 8M_{\text{sun}}$), which are not on this exam.

Part II: Problems

Tip: *Solve algebraically (using ratio/scaling methods) before plugging in numbers. This minimizes arithmetic errors and earns credit for correct setup even if the final number is off.*

1) Stellar Structure Under a Hypothetical Scenario. Suppose a main-sequence star doubles its mass while somehow keeping its central temperature T_c **fixed**. Assume the mean molecular weight μ and composition remain constant throughout.

(a) Determine how the star's radius R must change to keep T_c fixed. Express R_2 (after) in terms of R_1 (before). ***(4 pts)***

Solution:

Given: $M_2 = 2M_1$; μ, m_p, G, k_B unchanged; $T_{c,1} = T_{c,2}$.

Find: R_2/R_1.

Equation: $T_c \sim \frac{\mu G M m_p}{k_B R} \implies T_c \propto M/R$

Steps (ratio method):

$$\frac{T_{c,2}}{T_{c,1}} = \left(\frac{M_2}{R_2}\right)\left(\frac{M_1}{R_1}\right)^{-1} = \left(\frac{M_2}{M_1}\right)\left(\frac{R_1}{R_2}\right) = 1$$

Using $M_2/M_1 = 2$:

$$2 \cdot \left(\frac{R_1}{R_2}\right) = 1 \implies \frac{R_2}{R_1} = 2$$

$R_2 = 2\,R_1$ **— the radius must double.**

Unit check: T_c has units K on both sides; the ratio is dimensionless. ✓

Sanity check: Doubling M alone would **increase** T_c (hotter core). To cancel that effect and keep T_c fixed, the star must **expand** proportionally — which it does: R doubles. ✓

(b) How do the central pressure P_c and central density ρ_c change under this scenario? Express $P_{c,2}/P_{c,1}$ and $\rho_{c,2}/\rho_{c,1}$ as numerical ratios. ***(4 pts)***

Solution:

Equations (from formula sheet):

$$P_c \sim \frac{GM^2}{R^4} \implies P_c \propto M^2/R^4$$

$$\rho_c \sim \frac{M}{R^3} \implies \rho_c \propto M/R^3$$

Given: $M_2/M_1 = 2$, $R_2/R_1 = 2$ (from part (a)).

Steps:

$$\frac{P_{c,2}}{P_{c,1}} = \left(\frac{M_2}{M_1}\right)^2 \left(\frac{R_1}{R_2}\right)^4 = 2^2 \cdot \left(\frac{1}{2}\right)^4 = \frac{4}{16} = \frac{1}{4}$$

$$\frac{\rho_{c,2}}{\rho_{c,1}} = \left(\frac{M_2}{M_1}\right) \left(\frac{R_1}{R_2}\right)^3 = 2 \cdot \left(\frac{1}{2}\right)^3 = \frac{2}{8} = \frac{1}{4}$$

$$\boxed{P_{c,2} = \tfrac{1}{4}\,P_{c,1}, \quad \rho_{c,2} = \tfrac{1}{4}\,\rho_{c,1}}$$

Unit check: Both are ratios of like quantities, so dimensionless. ✓

Sanity check: A bigger, fluffier star at the same T_c has **lower** central pressure and density — consistent with the ideal-gas relation $P_c \sim \rho_c k_B T_c/(\mu m_p)$, where P_c and ρ_c scale together when T_c is held fixed. ✓

(c) Is this scenario physically realistic for a main-sequence star? If not, describe what a real star **actually** does when mass is gradually added — how do R, T_c, P_c, and the fusion rate respond? Justify your reasoning. ***(4 pts)***

Solution:

No, this scenario is not **physically realistic** for a main-sequence star.

Why the scenario fails: HSE requires the central pressure to support the column weight, which scales as $P_c \propto M^2/R^4$. Doubling M at fixed R would therefore demand P_c rise by $4\times$. But the hypothetical (which holds T_c fixed and forces R to double, part (a)) gives P_c **decreasing** by $4\times$ (part (b)) — leaving the core unable to support the extra mass against gravity.

What a real star does: the core **contracts** (opposite of the hypothetical). From the scalings:

$$M\uparrow,\ R\downarrow \implies \rho_c \propto M/R^3\uparrow \quad \text{and} \quad T_c \propto M/R\uparrow$$

So both ρ_c and T_c increase. This is a direct consequence of the virial theorem — **gravitational contraction heats the core** (the famous "negative heat capacity" of self-gravitating systems).

Fusion response: since the pp-chain rate scales steeply with temperature ($\varepsilon \propto \rho T^4$), even a modest rise in T_c dramatically accelerates fusion — which raises L until a new hydrostatic equilibrium is found, with the star settling at a new point higher up the main sequence. This self-regulation is precisely why fusion is stable: any perturbation that heats the core increases the luminosity, which lets the core radiate away the excess energy.

2) Fusion Sensitivity & The Radiation Limit. In this problem you will investigate two effects that together shape the upper end of the stellar mass distribution: the temperature sensitivity of hydrogen fusion and the limit set by radiation pressure.

(a) Consider the pp-chain hydrogen-fusion rate per unit mass, $\varepsilon_{\rm pp}$, deep in a stellar core. Suppose the core density **doubles** and the core temperature increases by 50% (i.e., $T \to 1.5T$). By what factor does $\varepsilon_{\rm pp}$ change? ***(4 pts)***

Solution:

Given: $\varepsilon_{\rm pp} \propto \rho T^4$; $\rho \to 2\rho$; $T \to 1.5T$.

Find: $\varepsilon_{\rm new}/\varepsilon_{\rm old}$.

Steps (ratio method):

$$\frac{\varepsilon_{\rm new}}{\varepsilon_{\rm old}} = \frac{(2\rho)(1.5T)^4}{\rho T^4} = 2 \cdot (1.5)^4 = 2 \cdot 5.0625 \approx 10.1$$

$\varepsilon_{\rm new} \approx 10\,\varepsilon_{\rm old}$ **— the fusion rate increases by a factor of** ~ 10**.**

Unit check: All factors are dimensionless ratios. ✓

Sanity check: Even a modest 50% temperature rise — with the density only doubling — increases fusion by an order of magnitude. The T^4 dependence is what makes stellar fusion exquisitely temperature-sensitive and self-regulating. ✓

(b) For a $M = 100M_{\rm sun}$ main-sequence star with a fully ionized hydrogen envelope, compute the ratio $L_{\rm MS}/L_{\rm Edd}$. Is this star super-Eddington ($L_{\rm MS} > L_{\rm Edd}$) or sub-Eddington ($L_{\rm MS} < L_{\rm Edd}$)? ***(4 pts)***

Solution:

Given: $M = 100 M_{\text{sun}}$; fully ionized H envelope (so κ is the same as the Sun's — κ_{es} — and cancels). From the formula sheet: $L_{\text{MS}} \propto M^{3.5}$, $L_{\text{Edd}} \propto M/\kappa$, $L_{\text{Edd,sun}} \approx 3.9 \times 10^4 L_{\text{sun}}$.

Find: $L_{\text{MS}}/L_{\text{Edd}}$ for this star.

Steps (ratio method): Combined scaling (at fixed κ):

$$\frac{L_{\text{MS}}}{L_{\text{Edd}}} \propto \frac{M^{3.5}}{M} = M^{2.5}$$

Using the Sun as reference:

$$\frac{(L_{\text{MS}}/L_{\text{Edd}})_{\text{star}}}{(L_{\text{MS}}/L_{\text{Edd}})_{\text{sun}}} = \left(\frac{M}{M_{\text{sun}}}\right)^{2.5} = (100)^{2.5} = 10^5$$

For the Sun:

$$(L_{\text{MS}}/L_{\text{Edd}})_{\text{sun}} = L_{\text{sun}}/L_{\text{Edd,sun}} = 1/(3.9 \times 10^4) \approx 2.6 \times 10^{-5}$$

Combining:

$$\left(\frac{L_{\text{MS}}}{L_{\text{Edd}}}\right)_{\text{star}} \approx 10^5 \times (2.6 \times 10^{-5}) \approx 2.6$$

$L_{\text{MS}}/L_{\text{Edd}} \approx 2.6 > 1$ **— the star is super-Eddington.**

Unit check: Ratio of luminosities is dimensionless. ✓

Sanity check: $L_{\text{MS}}/L_{\text{Edd}} \propto M^{2.5}$ grows rapidly with mass; the Sun is **deeply** sub-Eddington ($L_{\text{sun}}/L_{\text{Edd,sun}} \sim 10^{-5}$), but by $100 M_{\text{sun}}$ the ratio crosses unity — consistent with the observed upper stellar mass limit near ~ 100–$150 M_{\text{sun}}$. ✓

(c) Explain **why very massive stars ($M \gtrsim 100 M_{\text{sun}}$) are rare and short-lived.** Use your results from parts (a) and (b) to justify both claims quantitatively. ***(4 pts)***

Solution:

Massive stars are short-lived because fusion rates scale steeply with core temperature (pp-chain $\varepsilon \propto T^4$; CNO $\varepsilon \propto T^{17}$). Higher-mass stars have hotter cores, so they burn through their hydrogen fuel enormously faster than the Sun — $\tau_{\rm MS} \propto M^{-2.5}$ implies a $100 M_{\rm sun}$ star lives only $\sim 10\ \text{Gyr} \times (100)^{-2.5} \approx 100\ \text{kyr}$.

Massive stars are rare because they are **super-Eddington** (part (b)): radiation pressure on electron-scattering opacity exceeds gravity, driving violent line-driven winds that shed the envelope on timescales comparable to the nuclear lifetime. Above ~ 100–$150 M_{\rm sun}$, the star cannot hold itself together long enough to form in the first place — this sets the observed upper mass cutoff.

Together, the T-sensitivity of fusion and the Eddington limit conspire to make high-mass stars both cosmically rare **and** fleeting.

3) The Chandrasekhar Mass & White Dwarf Structure. When gravity fully compresses the electrons in a white dwarf until they become relativistic, a fundamental mass scale emerges that can be built from only three fundamental constants — $\hbar$, c, G — together with the mass of a proton m_p.

(a) Using **dimensional analysis**, construct a combination of $\hbar$, c, G, and m_p that has the units of mass and is expected to yield the Chandrasekhar mass scale. Evaluate your expression numerically (in grams and in M_{sun}). Show that it yields a value of order $\sim 1 M_{\text{sun}}$. ***(4 pts)***

Solution:

Given: $\hbar = 1.05 \times 10^{-27}$ erg s; $c = 3.0 \times 10^{10}$cm s^{-1}; $G = 6.67 \times 10^{-8}$cm^{3}g^{-1}s^{-2}; $m_p = 1.67 \times 10^{-24}$ g; $M_{\rm sun} = 2.0 \times 10^{33}$ g.

Find: A combination of these constants with units of mass [g].

Dimensional analysis: In base CGS:

$$[\hbar c] = \text{erg cm} = \text{g cm}^3\text{s}^{-2}$$

$$[G] = \text{cm}^3\text{g}^{-1}\text{s}^{-2}$$

$$[\hbar c/G] = \text{g cm}^3 \frac{\text{s}^{-2}}{\text{cm}^3} \text{g}^{-1}\text{s}^{-2} = \text{g}^2$$

So $\sqrt{\hbar c/G}$ has units of mass — this is the **Planck mass**, $M_{\rm Pl}$. To build a larger mass scale involving m_p, use the combination $M_{\rm Ch} \sim M_{\rm Pl}^3/m_p^2$:

$$M_{\rm Ch} \sim \frac{(\hbar c/G)^{3/2}}{m_p^2} \; [\text{g}]$$

Numerical evaluation:

$$\hbar c/G = \frac{3.15 \times 10^{-17} \text{ erg cm}}{6.67 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2}} = 4.72 \times 10^{-10}\text{g}^2$$

$$M_{\rm Pl} = \sqrt{4.72 \times 10^{-10}} \text{ g} = 2.17 \times 10^{-5} \text{ g}$$

$$M_{\rm Ch} \sim \frac{(2.17 \times 10^{-5}\text{ g})^3}{(1.67 \times 10^{-24}\text{ g})^2} = \frac{1.02 \times 10^{-14}\text{ g}^3}{2.79 \times 10^{-48}\text{ g}^2} \approx 3.7 \times 10^{33}\text{ g}$$

In solar units:

$$\frac{M_{\rm Ch}}{M_{\rm sun}} \approx \frac{3.7 \times 10^{33}}{2.0 \times 10^{33}} \approx 1.8$$

$$\boxed{M_{\rm Ch} \sim 3.7 \times 10^{33} \text{ g} \approx 1.8 M_{\rm sun}}$$

Unit check: $[M_{\rm Ch}] = \text{g}^3/\text{g}^2 = \text{g}$. ✓

Sanity check: The accepted value is $M_{\rm Ch} \approx 1.4 M_{\rm sun}$. Dimensional analysis is expected to match the order of magnitude but miss the $\mathcal{O}(1)$ prefactor (here, a factor involving the electron fraction Y_e and numerical coefficients). Agreement within $\sim 30\%$ is excellent. ✓

(b) Two (non-relativistic) white dwarfs have masses $M_1 = 0.6 M_{\rm sun}$ and $M_2 = 1.2 M_{\rm sun}$. Determine (i) which has the larger **radius** and by what factor, and (ii) which has the larger **average density** and by what factor. ***(4 pts)***

Solution:

Given: $M_1 = 0.6 M_{\text{sun}}$, $M_2 = 1.2 M_{\text{sun}}$. From the formula sheet: $R_{\text{WD}} \propto M^{-1/3}$ (non-relativistic degenerate matter); average density $\rho \propto M/R^3$.

Find: R_1/R_2 and ρ_1/ρ_2.

Steps — radius ratio (ratio method):

$$\frac{R_1}{R_2} = \left(\frac{M_1}{M_2}\right)^{-1/3} = \left(\frac{M_2}{M_1}\right)^{1/3} = (2)^{1/3} \approx 1.26$$

$R_1 \approx 1.26\, R_2$ **— the less-massive WD (**M_1**) has the larger radius.**

Steps — density ratio: Combine the scalings:

$$\rho \propto \frac{M}{R^3} \propto \frac{M}{\left(M^{-1/3}\right)^3} = \frac{M}{M^{-1}} = M^2$$

So $\rho_{\text{WD}} \propto M^2$ (more massive WDs are denser and also smaller). Then

$$\frac{\rho_1}{\rho_2} = \left(\frac{M_1}{M_2}\right)^2 = \left(\frac{0.6}{1.2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\rho_1 = \frac{1}{4}\rho_2$ **— the more-massive WD (**M_2**) is** $4\times$ **denser.**

Unit check: Both are dimensionless ratios of like quantities. ✓

Sanity check: The WD mass-radius relation is **inverted** — adding mass makes the star smaller. Since density piles up both from more mass **and** less volume, $\rho \propto M^2$ grows quickly. As $M \to M_{\text{Ch}} \approx 1.4 M_{\text{sun}}$, $\rho \to \infty$ in the non-relativistic limit — which is precisely where that limit breaks down (relativistic electrons, part (c)). ✓

(c) The Chandrasekhar mass $M_{\text{Ch}} \approx 1.4 M_{\text{sun}}$ is an **upper limit** on white-dwarf masses: no stable WD can exceed it. Explain **why this limit exists** physically, and justify your reasoning quantitatively. ***(4 pts)***

Solution:

As a WD is compressed, electrons are packed more tightly; by the uncertainty principle their momenta — and hence their speeds — grow. Below $\sim 1.4 M_{\text{sun}}$, electron speeds remain non-relativistic ($v \ll c$) and degeneracy pressure scales steeply as $P_{\text{deg}} \propto \rho^{5/3}$, rising fast enough to resist gravity.

When the density is so high that electrons become **relativistic** ($v \to c$), their speeds approach the cosmic speed limit even though their momenta can still increase. The pressure law therefore softens to $P_{\text{deg}} \propto \rho^{4/3}$. Crucially, **the rate at which P_{deg} grows with added mass can no longer outpace the rate at which gravity grows**: doubling the mass increases gravitational compression faster than the relativistic degeneracy pressure can respond. No stable equilibrium exists — the star collapses to a neutron star or black hole.

Physically, M_{Ch} marks the mass at which electrons are forced to become fully relativistic before they can support the star.

— End of Exam —

Formula sheet provided separately.