Math Survival Kit

ASTR 201 ▪ Tools of the Trade Companion

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1 Why this exists

Astronomy is full of numbers that are **too big**, **too small**, or **too many units** to handle by "gut feel" alone. In ASTR 201, you're expected to be able to:

- read and manipulate **scientific notation**
- use **exponent rules** fluently
- do **unit conversions** using factor-label (units-as-algebra)
- make **order-of-magnitude** estimates and sanity checks

Expectations (friendly but firm)

You are responsible for these math skills in this course.
That does **not** mean you already have them. It means you will build them, and you will practice them until they're reliable.

2 How to use this handout

1. Take the **Core diagnostic** below (~5 min). If you miss 2+, do the practice set.
2. Try the **Extended diagnostic** if you want to check roots/scaling fluency.
3. Keep this open while you do homework. It's a **reference**, not a punishment.

Section guide:

- **Core (used constantly):** scientific notation, exponents, PEMDAS, units, OOM
- **Extended (used often):** roots/fractional exponents, logarithms, proportionality

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3 Core diagnostic (~5 minutes)

Instructions: Work without a calculator if you can. Circle anything you're unsure about.

Scientific notation & exponents

1. Rewrite in standard decimal form: $3 \times 10^4 =$ ______________________________
2. Rewrite in scientific notation: $0.00072 =$ ______________________________
3. Compare: which is larger? 4×10^6 or 9×10^5 Answer: ______________________________
4. $10^3 \times 10^5 = 10$—
5. $10^{12}/10^7 = 10$—
6. $(10^4)^2 = 10$—
7. $2 \times 10^3 \times 5 \times 10^{-2} =$ ______________________________

Unit conversions

8. Convert: $3\,\text{km} =$ ______________________________ cm
9. Convert: $1\,\text{m}^2 =$ ______________________________ cm^2

OOM

10. Order of magnitude: 7×10^{10} rounds to 10—

Core diagnostic answer key

1. 30,000
2. 7.2×10^{-4}
3. 4×10^6 (bigger exponent wins)
4. 8
5. 5
6. 8
7. $(2 \cdot 5) \times 10^{3-2} = 10 \times 10^1 = 10^2$
8. 3×10^5 cm
9. 10^4 cm^2
10. 11 (rule of 3)

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4 Extended diagnostic (~5 minutes)

Order of operations & roots

11. Evaluate: $-2^4 =$ ____________________
12. Evaluate: $(-2)^4 =$ ____________________
13. $\sqrt{10^8} = 10^? \rightarrow$ ____________________
14. $(10^6)^{1/2} = 10^? \rightarrow$ ____________________

Proportionality

15. If $A \propto B^2$ and B triples, by what factor does A change? ____________________

Extended diagnostic answer key

11. -16 (exponent binds first, then negative)
12. 16 (parentheses make the base negative)
13. 4 (square root halves the exponent)
14. 3
15. $3^2 = 9$ (ninefold increase)

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5 Scientific notation (the language of astronomy)

5.1 What $a \times 10^n$ means

- a is the **coefficient** (usually $1 \leq a < 10$)
- 10^n tells you how many places the decimal moves
  - $n > 0$: big number (moves right)
  - $n < 0$: small number (moves left)

Example: $3.2 \times 10^5 = 320{,}000$

5.2 Normalizing (getting $1 \leq a < 10$)

- $32 \times 10^5 = 3.2 \times 10^6$
- $0.32 \times 10^5 = 3.2 \times 10^4$

Common mistake

Forgetting to change the exponent when you move the decimal.

5.3 Comparing numbers quickly

When comparing $a \times 10^n$ values: - **Exponent first** (bigger n wins) - If exponents match, compare coefficients

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6 Exponent rules you must know

These are not "math trivia." They're the rules that make astronomy manageable.

6.1 The Big Three

$$10^a \times 10^b = 10^{a+b}$$
$$\frac{10^a}{10^b} = 10^{a-b}$$
$$(10^a)^b = 10^{ab}$$

6.2 Negative exponents

$$10^{-3} = \frac{1}{10^3} = 0.001$$

6.3 Mixing coefficients and powers of ten

Treat it like: **(numbers)** $\times$ **(powers of ten)**.

Example:

$$(2 \times 10^3)(5 \times 10^{-2}) = (2 \cdot 5) \times 10^{3-2} = 10 \times 10^1 = 10^2$$

> **Sanity trick**
>
> If you get $a \geq 10$, rewrite (normalize) so the coefficient is between 1 and 10.

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7 Order of operations (PEMDAS)

When you see $F = GMm/r^2$, how do you parse it?

7.1 The hierarchy

Parentheses $\rightarrow$ **E**xponents $\rightarrow$ **M**ultiplication/**D**ivision $\rightarrow$ **A**ddition/**S**ubtraction

Operations at the same level go **left to right**.

7.2 The traps that catch students

Trap 1: Negatives and exponents

$$-3^2 = -(3^2) = -9 \qquad \text{but} \qquad (-3)^2 = 9$$

The exponent binds tighter than the negative sign unless you use parentheses.

Trap 2: Division chains

The expression a/bc is ambiguous. It could mean either:

$$\frac{a}{bc} \qquad \text{or} \qquad \left(\frac{a}{b}\right)c = \frac{ac}{b}$$

Never write it that way. Use parentheses or a stacked fraction.

Trap 3: Fractions of fractions

$$\frac{a/b}{c} = \frac{a}{bc} \qquad \text{and} \qquad \frac{a}{b/c} = \frac{ac}{b}$$

Dividing by a fraction = multiplying by its reciprocal.

7.3 Astronomy example

Parse $F = GMm/r^2$:

1. Exponent first: r^2
2. Then multiplication/division left to right: $G \cdot M \cdot m/r^2$
3. Result: $F = \dfrac{GMm}{r^2}$

The fix

When in doubt, add parentheses. They cost nothing and prevent errors.

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8 Fractional exponents and roots

Roots are just exponents in disguise.

8.1 The connection

$$a^{1/2} = \sqrt{a} \qquad a^{1/3} = \sqrt[3]{a} \qquad a^{1/n} = \sqrt[n]{a}$$

8.2 The power rule still works

$$(10^6)^{1/2} = 10^{6\times 1/2} = 10^3$$

Translation: Taking a square root **halves the exponent**.

8.3 Kepler's Law uses this constantly

$$P \propto r^{3/2} = r^{1.5} = r \cdot \sqrt{r}$$

If r doubles:

$$P_{\text{new}} = P_{\text{old}} \times 2^{3/2} = P_{\text{old}} \times 2\sqrt{2} \approx 2.8\, P_{\text{old}}$$

8.4 Breaking apart roots

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Example: Simplify $\sqrt{\dfrac{r^3}{GM}}$

$$\sqrt{\frac{r^3}{GM}} = \frac{\sqrt{r^3}}{\sqrt{GM}} = \frac{r^{3/2}}{\sqrt{G}\sqrt{M}}$$

The pattern

To take a square root of 10^n: divide the exponent by 2.
$\sqrt{10^6} = 10^3$, $\sqrt{10^{33}} = 10^{16.5} \approx 3 \times 10^{16}$

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9 Unit conversions: factor-label method (units as algebra)

9.1 The core idea

A conversion factor is just **multiplying by 1**.

If $1\,\text{m} = 100\,\text{cm}$, then both are true:

$$\frac{100\,\text{cm}}{1\,\text{m}} = 1 \qquad \text{and} \qquad \frac{1\,\text{m}}{100\,\text{cm}} = 1$$

9.2 The template

1. Write the quantity with units.
2. Multiply by conversion factors so the unwanted unit cancels.
3. Cancel units explicitly.
4. Combine powers of ten.

Example: $30\,\mathrm{km/s} \to \mathrm{cm/s}$

$$30\,\frac{\mathrm{km}}{\mathrm{s}} \times \frac{10^3\,\mathrm{m}}{1\,\mathrm{km}} \times \frac{10^2\,\mathrm{cm}}{1\,\mathrm{m}} = 30 \times 10^5\,\frac{\mathrm{cm}}{\mathrm{s}} = 3 \times 10^6\,\mathrm{cm/s}$$

The #1 unit mistake

Dropping units mid-calculation.
Rule: If units disappear before the last line, something went wrong.

9.3 Squared and cubed units (the sneaky trap)

If $1\,\mathrm{m} = 10^2\,\mathrm{cm}$, then:

$$1\,\mathrm{m}^2 = (10^2\,\mathrm{cm})^2 = 10^4\,\mathrm{cm}^2$$
$$1\,\mathrm{m}^3 = (10^2\,\mathrm{cm})^3 = 10^6\,\mathrm{cm}^3$$

Translation: You must raise the conversion factor to the same power.

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10 Order-of-magnitude (OOM) thinking

Astronomy often rewards being **roughly right**.

10.1 Rule of 3 (fast rounding)

When estimating: - coefficient $< 3 \to$ round down to **1** - coefficient $> 3 \to$ round up to **10**

Examples: - $2 \times 10^{33} \approx 10^{33}$ - $7 \times 10^{10} \approx 10^{11}$

10.2 What OOM is (and isn't)

- OOM means you care about the **exponent** most.
- Being off by a factor of **2–3** is often fine.
- Being off by a factor of **10^8** means your model or units are broken.

The sanity-check question

"What real-world scale should this live near?"
If your result puts a star at 10^{11} cm, that's **Sun-sized**, not star-distance.

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11 Logarithms (just enough)

Logarithms extract exponents. That's the whole idea.

11.1 The definition

$$\log_{10}(10^n) = n$$

If $x = 10^n$, then $\log_{10}(x) = n$.

11.2 Why astronomers care

1. **"Order of magnitude" literally means "the log."** When we say two quantities differ by 3 orders of magnitude, we mean $\log_{10}(A/B) = 3$.
2. **Magnitudes use logs.** The brightness scale:

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

3. **Log scales compress huge ranges.** A plot from 10^{-13} to 10^{28} cm is impossible on a linear axis but simple on a log axis.

11.3 The rules you need

Rule	Formula	Example
Log of a product	$\log(ab) = \log a + \log b$	$\log(10^3 \times 10^5) = 3 + 5 = 8$
Log of a quotient	$\log(a/b) = \log a - \log b$	$\log(10^{12}/10^7) = 12 - 7 = 5$
Log of a power	$\log(a^n) = n \log a$	$\log(10^{3\times 2}) = 6$

You won't need to compute logs by hand

Calculators handle the numbers. You need to understand what logs *mean* and why they appear in formulas.

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12 Proportionality notation

The symbol $\propto$ means "is proportional to." It's everywhere in physics.

12.1 What it means

$$A \propto B \quad \Longleftrightarrow \quad A = k \cdot B \text{ for some constant } k$$

"A is proportional to B" means if you double B, A doubles too.

12.2 What it does NOT mean

$\propto$ does **not** tell you the value of the constant k. It only tells you the **relationship**.

12.3 The power of ratios

If $A \propto B^n$, then for two systems:

$$\frac{A_2}{A_1} = \left(\frac{B_2}{B_1}\right)^n$$

The constant k **cancels**. You don't need to know it.

12.4 Examples from ASTR 201

Relationship	Meaning	If input doubles…
$P \propto r^{3/2}$	Period scales with radius	P increases by $2^{1.5} \approx 2.8\times$
$F \propto 1/r^2$	Inverse-square law	F decreases by $4\times$
$L \propto R^2 T^4$	Stefan-Boltzmann	Double $T \rightarrow L$ increases $16\times$
$R_s \propto M$	Schwarzschild radius	R_s doubles

The workflow

1. Write down the proportionality.
2. Form a ratio between two cases.
3. Cancel constants and compute.

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13 Calculator survival (optional, but useful)

You do not need a fancy calculator, but you do need to avoid common traps.

- Use the **EE/EXP** key for scientific notation input.
  - 3.8×10^{26} should be typed like `3.8 EE 26`.
- Use **parentheses** aggressively.
- After a calculation, ask: **does the exponent make sense?**

Common calculator failure mode

Typing 3.8×10^{26} as 3.8×10 then $+26$.
That is not how exponents work. (The calculator will happily let you be wrong.)

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14 Quick reference tables

14.1 SI prefixes (the ones we actually use)

Prefix	Symbol	Power
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}

14.2 High-use conversions

Conversion	Value
1 km	10^{5} cm
1 m	10^{2} cm
1 kg	10^{3} g
1 J	10^{7} erg
1 W	10^{7} erg/s

14.3 Course anchor values (CGS where appropriate)

Quantity	Approx value
Speed of light c	3×10^{10} cm/s
Gravitational constant G	$6.67 \times 10^{-8}\ \mathrm{cm^3\ g^{-1}\ s^{-2}}$
1 AU	1.50×10^{13} cm
1 pc	3.09×10^{18} cm
Solar mass $M_\odot$	2.0×10^{33} g
Solar radius $R_\odot$	7.0×10^{10} cm
Solar luminosity $L_\odot$	3.8×10^{33} erg/s

Reality check

These values are the “mental yardsticks” you’ll use all semester. You do **not** need to memorize everything at once.

15 Practice set

15.1 Level 0 — warm-up (scientific notation + exponents)

1. Normalize: $45 \times 10^6 \rightarrow$ ____________________________
2. Normalize: $0.008 \times 10^5 \rightarrow$ ____________________________
3. Compute: $10^{-3} \times 10^8 = 10$ —
4. Compute: $10^{12}/10^{-4} = 10$ —
5. Compute: $(10^2)^5 = 10$ —

15.2 Level 1 — order of operations and roots

6. Evaluate: $-5^2 =$ ____________________________
7. Evaluate: $(-5)^2 =$ ____________________________
8. Simplify: $\frac{12/4}{3} =$ ____________________________
9. Simplify: $\sqrt{10^{12}} = 10^? \rightarrow$ ____________________________
10. Compute: $(10^9)^{1/3} = 10^? \rightarrow$ ____________________________
11. If $P \propto r^{3/2}$ and $r = 4$, what is $r^{3/2}$? ____________________________

15.3 Level 2 — unit conversions (factor-label)

12. Convert 12 km to cm.
13. Convert 0.35 m to cm.
14. Convert 250 m/s to cm/s.
15. Convert $2.0\,\mathrm{m}^2$ to cm^2.
16. Convert 5 J to erg.

15.4 Level 3 — astronomy-flavored problems

17. The Sun's luminosity is 3.8×10^{26} W. Convert to erg/s.
18. OOM round: 2.7×10^{33} g $\rightarrow$ ____________________________
19. OOM round: 9.1×10^{-28} g $\rightarrow$ ____________________________
20. If $R_s \propto M$ and $R_s(1\,M_\odot) \approx 3\,\mathrm{km}$, estimate $R_s(10\,M_\odot)$.
21. A quantity comes out to 4×10^{11} cm. Is that closer to an Earth radius ($\sim 10^9$ cm) or a solar radius ($\sim 10^{11}$ cm)? Explain in one sentence.

15.5 Level 4 — proportionality and scaling

22. If $L \propto R^2$ and a star's radius doubles, by what factor does its luminosity change?
23. If $F \propto 1/r^2$ and distance triples, by what factor does flux change?
24. Using $P \propto r^{3/2}$: Earth orbits at 1 AU with $P = 1$ yr. Mars orbits at 1.52 AU. Estimate Mars's period.
25. The Schwarzschild radius scales as $R_s \propto M$. Sgr A* has mass $4 \times 10^6\,M_\odot$. If $R_s(1\,M_\odot) \approx 3\,\text{km}$, estimate R_s for Sgr A* in km.

Practice answer key

Level 0

1. 4.5×10^7
2. 8×10^2
3. 5
4. 16
5. 10

Level 1

6. -25 (exponent binds before negative)
7. 25
8. 1 (it's $(12/4)/3 = 3/3 = 1$)
9. 6 (square root halves the exponent: $\sqrt{10^{12}} = 10^6$)
10. 3 (cube root divides by 3: $(10^9)^{1/3} = 10^3$)
11. $4^{3/2} = 4 \cdot \sqrt{4} = 4 \cdot 2 = 8$

Level 2

12. $12 \times 10^5 = 1.2 \times 10^6\,\text{cm}$
13. $35\,\text{cm}$
14. $250 \times 10^2 = 2.5 \times 10^4\,\text{cm/s}$
15. $2.0 \times 10^4\,\text{cm}^2$
16. $5 \times 10^7\,\text{erg}$

Level 3

17. $3.8 \times 10^{33}\,\text{erg/s}$
18. $10^{33}\,\text{g}$ (rule of 3)
19. $10^{-27}\,\text{g}$ (rule of 3)
20. $\approx 30\,\text{km}$
21. Solar-radius scale (10^{11} cm) — exponent matches Sun scale.

Level 4

22. $2^2 = 4$ (luminosity quadruples)
23. $1/3^2 = 1/9$ (flux decreases by factor of 9)
24. $P^2 = 1.52^3 \approx 3.5$, so $P \approx \sqrt{3.5} \approx 1.9$ years
25. $R_s = 3\,\text{km} \times 4 \times 10^6 = 1.2 \times 10^7\,\text{km}$

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16 What "show your work" means in ASTR 201

When math is part of the physics, your work needs to be readable—and **checkable**.

16.1 Why this matters

Small mistakes are easy to make: a dropped exponent, a flipped fraction, a unit that didn't cancel. The only reliable way to catch them is to **write every step explicitly**.

If you skip steps, you can't find your error. Neither can we.

16.2 The template

1. **Write the equation first** — before plugging in numbers. This separates "what physics am I using?" from "what numbers go where?"
2. **Carry units through every step** — not just at the start and end. Units that don't cancel are a red flag.
3. **Show exponent arithmetic explicitly** — write $10^{26} \times 10^7 = 10^{33}$, not just the answer.
4. **Simplify before you calculate** — cancel terms, combine exponents, reduce fractions *on paper* first. The calculator is the last step, not the first.
5. **Box the final answer with units** — make it easy to find.
6. **Sanity check** — one sentence. Does the magnitude make sense?

Calculator errors are sneaky

Typing mistakes are easy to make and hard to spot. A misplaced parenthesis or a wrong exponent key can silently wreck your answer.
Always check your calculator work. Re-enter the calculation a second way, or estimate the answer by hand first so you know what ballpark to expect. If the calculator says 10^{47} but your estimate says 10^{33}, trust the estimate—you probably made a typo.

The hard truth

Most errors aren't conceptual—they're mechanical. A sign flip. A forgotten square. A unit that should have been squared but wasn't.
Showing your work isn't about proving you know the material. It's about building a trail you can follow when something goes wrong. And something *will* go wrong.

The real skill

You're not being graded on "being fast." You're being graded on **being trustworthy**—and trustworthy work can be checked.