

**Electromagnetic Radiation**

$$c = \lambda\nu \quad E = h\nu = \frac{hc}{\lambda}$$

**Planck Function**

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/k_B T} - 1}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda k_B T} - 1}$$

**Wien's Displacement Law**

$$\lambda_{\text{peak}} T = 2.898 \times 10^6 \text{ nm} \cdot \text{K}$$

**Stefan-Boltzmann**

$$F_\star = \sigma T_{\text{eff}}^4 \quad L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

**Planetary Equilibrium Temp ( $A = \text{albedo}$ )**

$$T_{\text{eq}} = \left( \frac{L_\star(1-A)}{16\pi\sigma d^2} \right)^{1/4}$$

**Brightness, Magnitudes, Distance**

$$F = \frac{L}{4\pi d^2}$$

(apparent flux)

$$m_1 - m_2 = -2.5 \log_{10}(F_1/F_2)$$

$$m - M = 5 \log_{10}(d/10\text{pc})$$

$$M = 4.83 - 2.5 \log_{10}(L/L_\odot)$$

$$d[\text{pc}] = 1/p[\text{arcsec}]$$

$$1 \text{ rad} = 206,265 \text{ arcsec}$$

**Kirchhoff's Spectroscopy Laws**

(1) Hot dense source  $\rightarrow$  continuous. (2) Hot low-density gas  $\rightarrow$  emission lines. (3) Cool gas before hot source  $\rightarrow$  absorption lines.

**Gravity, Energy, Virial**

$$F_g = \frac{Gm_1 m_2}{r^2} \quad g = \frac{GM}{R^2}$$

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}} \quad v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

$$E_K = \frac{1}{2}mv^2 \quad U = -\frac{GMm}{r}$$

$$E = E_K + U \quad (E < 0 \text{ bound})$$

**Virial (bound):**

$$2E_K + U = 0$$

**Binding-energy scale:**

$$E_g \sim -GM^2/R$$

**Bound orbit:**

$$E = -GMm/(2a)$$

**Kepler & Binary Orbits**

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

$$\frac{M_1 + M_2}{M_\odot} = \frac{(a/\text{AU})^3}{(P/\text{yr})^2}$$

**Spec. binary:**

$$K_1/K_2 = M_2/M_1$$

**Center of mass:**  $M_1 a_1 = M_2 a_2$ ,  $a = a_1 + a_2$ .

**Main-Sequence Scalings**

$$L \propto M^{3.5}, \quad t_{\text{MS}} \propto M/L \propto M^{-2.5}$$

**Doppler & Telescopes (non-rel.,  $v \ll c$ )**

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

$$A_{\text{coll}} = \pi D^2/4 \quad \alpha_\lambda = 1.22\lambda/D \quad R = \lambda/\Delta\lambda$$

**Stellar Structure Equations**

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

(mass continuity)

$$\frac{dP}{dr} = -\rho \frac{GM(r)}{r^2}$$

(HSE)

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

(energy gen.)

$$\frac{dT}{dr} = -\frac{3\kappa\rho L(r)}{16\pi acT^3 r^2}$$

(rad. transp.)

**One-Zone Scalings**

$$P_c \sim \frac{GM^2}{R^4} \quad \rho_c \sim M/R^3$$

$$T_c \sim \frac{\mu GM m_p}{k_B R} \quad g_{\text{surf}} \sim (GM)/R^2$$

$$E_g \sim -(GM^2)/R$$

**Toy MS scalings (radiative stars):**

$$L \propto M^3/\kappa \quad R \propto M^{3/7}$$

**Pressure Laws**

$$P_{\text{gas}} = nk_B T = \frac{\rho k_B T}{\mu m_p}$$

$$P_{\text{rad}} = \left(\frac{1}{3}\right) a T^4 \quad (a = 4\sigma/c)$$

$$P_{\text{deg, NR}} \propto \rho^{5/3}/m_e \quad P_{\text{deg, R}} \propto \rho^{4/3}$$

$$P_{\text{tot}} = P_{\text{gas}} + P_{\text{rad}} + P_{\text{deg}}$$

**Mean mol. weight (ionized):**

$$1/\mu \approx 2X + (3/4)Y + (1/2)Z$$

solar  $\mu \approx 0.6$ , pure-H  $\mu \approx 0.5$ .

**Stellar Timescales**

$$t_{\text{dyn}} \sim \sqrt{R^3/GM} \sim 1/\sqrt{G\bar{\rho}}$$

$$t_{\text{KH}} \sim |E_g|/L \sim \frac{GM^2}{RL}$$

$$t_{\text{nuc}} \sim \frac{\epsilon_{\text{nuc}} f M c^2}{L}$$

$f \sim 0.1$  (fusible mass fraction for H);  $\epsilon_{\text{nuc}} \approx 0.007$  (H-fusion efficiency).

**Nuclear Fusion**
**H fusion:**

$${}^4\text{H} \rightarrow {}^4\text{He}$$

$$\Delta E \approx 26.7 \text{ MeV}, \quad \epsilon_{\text{H}} \approx 0.007.$$

$$\epsilon_{\text{pp}} \propto \rho T^4 \quad \epsilon_{\text{CNO}} \propto \rho T^{17}$$

**Triple- $\alpha$ :**

$$3 {}^4\text{He} \rightarrow {}^{12}\text{C} + 7.3 \text{ MeV}; \quad \epsilon_{3\alpha} \propto \rho^2 T^{40}$$

**Coulomb Barrier & Tunneling**

$$U_{\text{Coul}}(r) = \frac{Z_1 Z_2 e^2}{r} \quad \lambda_{\text{dB}} = h/p$$

$$\Gamma \approx e^{-2G}; \quad G \propto (Z_1 Z_2)/\sqrt{T}$$

(Gamow)

**Radiation Transport**

$$\ell = 1/(\kappa\rho) \quad d\tau = \kappa\rho dr$$

$$t_{\text{diff}} \sim R^2/(\ell c) = (\kappa\rho R^2)/c$$

$\kappa_{\text{es}} = 0.34 \text{ cm}^2 \text{ g}^{-1}$  (H-rich).

**Eddington Luminosity**

$$L_{\text{Edd}} = \frac{4\pi GMc}{\kappa} \quad L_{\text{Edd}, \odot} \approx 3.9 \times 10^4 L_\odot$$

**Quantum Mechanics**

$$\Delta p \Delta x \gtrsim \hbar/2 \quad \lambda_{\text{dB}} = h/p$$

$$\hbar = h/(2\pi)$$

### Compact Objects

#### White Dwarfs

$$R_{\text{WD}} \propto M^{-1/3} \quad R_{\text{WD}} \approx (0.01 R_{\odot})(M_{\text{WD}}/0.7 M_{\odot})^{-1/3}$$

$$M_{\text{Ch}} \approx 1.4 M_{\odot}$$

**Neutron Stars**  $R_{\text{NS}} \approx 10\text{--}15 \text{ km} \sim 10^{-5} R_{\odot}$       $M_{\text{TOV}} \approx 2\text{--}3 M_{\odot}$

#### Black Holes — Schwarzschild Radius

$$R_S = \frac{2GM}{c^2} \quad R_S / \text{km} \approx 2.95 (M/M_{\odot}) \quad C \equiv R_S/R$$

**Stellar Mass Limits**  $M_{\text{min}} \approx 0.08 M_{\odot}$       $M_{\text{max}} \sim 100\text{--}150 M_{\odot}$

### Galaxies & Cosmology

#### Enclosed Mass from Rotation Curve

$$v^2 = \frac{GM(<r)}{r} \implies M(<r) = \frac{rv^2}{G}$$

#### Hubble's Law (low $z$ )

$$v = H_0 d \quad H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

#### Cosmological Redshift & Scale Factor

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \frac{a_0}{a_{\text{emit}}}$$

With  $a_0 = 1$ :  $a_{\text{emit}} = 1/(1+z)$ .

**Cosmic Cooling (from Wien + redshift)** For thermal radiation that redshifts with the universe:

$$\frac{T_{\text{emit}}}{T_{\text{obs}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = 1 + z$$

CMB today:  $T_0 \approx 2.73 \text{ K}$ .

**Matter Density Scaling** Mass conserved, volume  $\propto a^3$ :

$$\rho_m \propto a^{-3} = (1+z)^3$$

### Math Reference

**Order of Operations — PEMDAS** Parentheses → Exponents → Multiplication / Division (L → R) → Addition / Subtraction (L → R)

#### Exponents

$$y^a y^b = y^{a+b}; (y^a)^b = y^{ab}; y^{-a} = 1/y^a; y^{1/n} = \sqrt[n]{y}$$

#### Logarithms

$$\log(ab) = \log a + \log b; \log(a/b) = \log a - \log b$$

$$\log(a^b) = b \log a; \log_{10} 10^a = a; \ln e^a = a$$

#### Sphere Geometry

$$V = \frac{4}{3}\pi r^3 \quad A_s = 4\pi r^2 \quad A_p = \pi r^2 \quad C = 2\pi r$$

#### Useful Approximations (small $x$ , small $\theta$ )

$$\sin \theta \approx \theta \quad (1+x)^n \approx 1+nx \quad e^x \approx 1+x$$

### Ratio Method

For  $Q \propto A^a B^b$ :

$$Q_2/Q_1 = (A_2/A_1)^a (B_2/B_1)^b$$

Use **before** plugging in numbers — units and prefactors cancel.

### Astronomical Values & Conversions

$M_{\odot}$	$2.0 \times 10^{33} \text{ g}$
$R_{\odot}$	$7.0 \times 10^{10} \text{ cm}$
$L_{\odot}$	$3.8 \times 10^{33} \text{ erg s}^{-1}$
$T_{\text{eff},\odot}$	5800 K
$t_{\text{MS},\odot}$	10 Gyr
$M_{\text{Jup}}, R_{\text{Jup}}$	$1.9 \times 10^{30} \text{ g}, 7.0 \times 10^9 \text{ cm}$
$M_{\oplus}, R_{\oplus}$	$6.0 \times 10^{27} \text{ g}, 6.4 \times 10^8 \text{ cm}$
1 AU	$1.496 \times 10^{13} \text{ cm}$
1 pc	$3.086 \times 10^{18} \text{ cm} \approx 3.26 \text{ ly}$
1 kpc	$3.086 \times 10^{21} \text{ cm} = 10^3 \text{ pc}$
1 Mpc	$3.086 \times 10^{24} \text{ cm} = 10^6 \text{ pc}$
1 ly	$9.46 \times 10^{17} \text{ cm}$
1 yr	$3.156 \times 10^7 \text{ s}$
$T_{\text{CMB},0}$	2.73 K (present-day CMB)
$H_0$	$70 \text{ km s}^{-1} \text{ Mpc}^{-1}$

### Physical Constants (CGS)

Quantity	Value
$c$	$3.0 \times 10^{10} \text{ cm s}^{-1}$
$G$	$6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$
$G$ (galaxy units)	$4.30 \times 10^{-6} \text{ kpc (km/s)}^2 M_{\odot}^{-1}$
$k_B$	$1.38 \times 10^{-16} \text{ erg K}^{-1}$
$\sigma$	$5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$
$a = 4\sigma/c$	$7.57 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$
$h$	$6.63 \times 10^{-27} \text{ erg s}$
$\hbar$	$1.05 \times 10^{-27} \text{ erg s}$
$m_p$	$1.67 \times 10^{-24} \text{ g}$
$m_n$	$1.68 \times 10^{-24} \text{ g}$
$m_e$	$9.11 \times 10^{-28} \text{ g}$
amu	$1.66 \times 10^{-24} \text{ g}$
$e$	$4.80 \times 10^{-10} \text{ esu}$
1 eV	$1.60 \times 10^{-12} \text{ erg}$
$hc$	1240 eV nm
$\kappa_{\text{es}}$	$0.34 \text{ cm}^2 \text{ g}^{-1}$

### CGS Derived Units

Quantity	Unit	Base units
Force	dyne	$\text{g cm s}^{-2}$
Energy	erg	$\text{g cm}^2 \text{ s}^{-2}$
Power	$\text{erg s}^{-1}$	$\text{g cm}^2 \text{ s}^{-3}$
Pressure	$\text{dyne cm}^{-2}$	$\text{g cm}^{-1} \text{ s}^{-2}$
Density	$\text{g cm}^{-3}$	—
Charge	esu	$\text{g}^{1/2} \text{ cm}^{3/2} \text{ s}^{-1}$

### Powers-of-Ten Prefixes

n (nano)	$10^{-9}$	k (kilo)	$10^3$
$\mu$ (micro)	$10^{-6}$	M (mega)	$10^6$
m (milli)	$10^{-3}$	G (giga)	$10^9$
c (centi)	$10^{-2}$	T (tera)	$10^{12}$