Astronomy 201 Formula Sheet

Midterm 1 · Modules 1 & 2 · CGS Units

Electromagnetic Radiation

Speed of Light

$$c = \lambda\,\nu$$

λ [cm] = wavelength $\qquad$ ν [Hz = s^{-1}] = frequency

Photon Energy

$$E = h\nu = \frac{hc}{\lambda}$$

Planck Function (Blackbody Spectrum)

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{\,h\nu/k_BT} - 1} \;\; [\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}\ sr^{-1}}]$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{\,hc/\lambda k_BT} - 1} \;\; [\mathrm{erg\ s^{-1}\ cm^{-3}\ sr^{-1}}]$$

Wien's Displacement Law

$$\lambda_{\mathrm{peak}} = \frac{2.898 \times 10^6\ \mathrm{nm{\cdot}K}}{T}$$

Stefan-Boltzmann Law

$$F_\star = \sigma\,T_{\mathrm{eff}}^4 \;\; [\mathrm{erg\ s^{-1}\ cm^{-2}}]$$

$$L = 4\pi\,R^2\,\sigma\,T_{\mathrm{eff}}^4 \;\; [\mathrm{erg\ s^{-1}}]$$

Planetary Equilibrium Temperature

$$T_{\mathrm{eq}} = \left(\frac{L_\star\,(1-A)}{16\pi\,\sigma\,d^2}\right)^{1/4}$$

A = albedo

Brightness & Magnitudes

Apparent Brightness (Observed Flux)

$$F = \frac{L}{4\pi\,d^2} \;\; [\mathrm{erg\ s^{-1}\ cm^{-2}}]$$

Magnitude System

$$m_1 - m_2 = -2.5\,\log_{10}\!\left(\frac{F_1}{F_2}\right)$$

Distance Modulus

$$m - M = 5\,\log_{10}\!\left(\frac{d}{10\ \mathrm{pc}}\right)$$

Absolute Magnitude from Luminosity

$$M = 4.83 - 2.5\,\log_{10}\!\left(\frac{L}{L_\odot}\right)$$

where $M_{V,\odot} = 4.83$.

Angular Size & Parallax

Small-Angle Approximation

$$\alpha = \frac{s}{d} \;\; [\mathrm{rad}]$$

s = physical size $\qquad$ d = distance

1 rad = 206,265 arcsec

Trigonometric Parallax

$$d = \frac{b}{p}\,; \quad d\ [\mathrm{pc}],\ p\ [\mathrm{arcsec}],\ b\ [\mathrm{AU}]$$

Kirchhoff's Laws of Spectroscopy

1. Hot, dense source → **continuous spectrum**
2. Hot, low-density gas → **emission lines**
3. Cool gas before hot source → **absorption lines**

Gravity

Newton's Second Law

$$F = m\,a \;\; [\mathrm{dyne = g\ cm\ s^{-2}}]$$

Gravitational Force

$$F_g = \frac{G\,m_1\,m_2}{r^2}$$

r = center-to-center separation

Surface Gravity

$$g = \frac{GM}{R^2} \;\; [\mathrm{cm\ s^{-2}}]$$

Orbital Speed ($m \ll M$, circular)

$$v_{\mathrm{orb}} = \sqrt{\frac{GM}{r}} \;\; [\mathrm{cm\ s^{-1}}]$$

Escape Speed

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}} \;\; [\mathrm{cm\ s^{-1}}]$$

Center of Mass

$$M_1\,a_1 = M_2\,a_2, \qquad a = a_1 + a_2$$

a_1, a_2 = distances of M_1, M_2 from center of mass

Energy & Virial Theorem

Kinetic Energy

$$E_K = \frac{1}{2}\,m\,v^2 \;\; [\mathrm{erg}]$$

Gravitational Potential Energy

$$U(r) = -\frac{GMm}{r} \qquad (U \to 0 \text{ as } r \to \infty)$$

Total Energy

$$E = E_K + U$$

$E < 0$: bound $\qquad$ $E = 0$: escape $\qquad$ $E > 0$: unbound

Virial Theorem (Bound Equilibrium)

$$2\,E_K + U = 0$$

Kepler's Laws & Binary Orbits

Kepler's Third Law (Newton's Form)

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)}\,a^3$$

P [s] = orbital period $\qquad$ a [cm] = semi-major axis

Convenient Solar-Unit Form

$$\frac{M_1 + M_2}{M_\odot} = \frac{(a/\mathrm{AU})^3}{(P/\mathrm{yr})^2}$$

Spectroscopic Binary Velocities

$$\frac{K_1}{K_2} = \frac{M_2}{M_1}$$

K_1, K_2 = velocity amplitudes

Mass-Luminosity Relation (Main Sequence)

$$L \propto M^{3.5}$$

Main-Sequence Lifetime

$$t_{\rm MS} \propto \frac{M}{L} \propto M^{-2.5}$$

Bound Orbit Energy

$$E = -\frac{GMm}{2a}$$

a = semi-major axis

Inferring Motion

Doppler Effect (non-relativistic, $v_r \ll c$)

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

λ_0 = rest wavelength $\lambda_{\rm obs}$ = observed wavelength

Telescopes

Collecting Area

$$A = \frac{\pi}{4} D^2$$

D = aperture diameter

Angular Resolution (Diffraction Limit)

$$\alpha_\lambda = 1.22 \, \frac{\lambda}{D} \ \text{[rad]}$$

Spectral Resolution

$$R = \frac{\lambda}{\Delta\lambda}$$

Mathematical Formulae

Powers

$$y^a \cdot y^b = y^{a+b} \qquad y^a / y^b = y^{a-b}$$
$$(y^a)^b = y^{ab} \qquad y^{1/n} = \sqrt[n]{y}$$
$$y^{-a} = 1/y^a \qquad y^0 = 1$$

Logarithms

$$\log(a \times b) = \log a + \log b$$
$$\log(a/b) = \log a - \log b$$
$$\log(a^b) = b \, \log a$$

$x = 10^a \Leftrightarrow \log_{10} x = a$ $y = e^b \Leftrightarrow \ln y = b$

Sphere Geometry

$V = \frac{4}{3}\pi r^3$ (volume) $A_s = 4\pi r^2$ (surface area)

$A_p = \pi r^2$ (cross-section) $C = 2\pi r$ (circumference)

Physical Constants (CGS)

Speed of light	$c = 3.0 \times 10^{10}$ cm s^{-1}
	$= 3.0 \times 10^5$ km s^{-1}
Gravitational const.	$G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}
Boltzmann const.	$k_B = 1.38 \times 10^{-16}$ erg K^{-1}
Stefan-Boltzmann	$\sigma = 5.67 \times 10^{-5}$
	erg cm^{-2} s^{-1} K^{-4}
Planck's const.	$h = 6.63 \times 10^{-27}$ erg s
Proton mass	$m_p \approx 1.67 \times 10^{-24}$ g
Electron mass	$m_e = 9.11 \times 10^{-28}$ g
1 eV	$= 1.60 \times 10^{-12}$ erg
hc	$= 1{,}240$ eV·nm

Astronomical Values & Conversions

Solar Values

$M_\odot$	2.0×10^{33} g
$R_\odot$	7.0×10^{10} cm
$T_{\rm eff,\,\odot}$	5800 K
$L_\odot$	3.8×10^{33} erg s^{-1}

Planetary Values

$M_{\rm Jup}$	1.9×10^{30} g
$R_{\rm Jup}$	7.0×10^9 cm
$M_\oplus$	6.0×10^{27} g
$R_\oplus$	6.4×10^8 cm

Distances

1 pc	3.086×10^{18} cm = 3.26 ly
1 ly	9.46×10^{17} cm
1 AU	1.496×10^{13} cm
1 yr	3.156×10^7 s

Angular Measure

1°	60 arcmin
1′	60 arcsec
1 rad	206,265 arcsec

CGS base units	cm, g, s	
1 dyne (force)	g cm s^{-2}	
1 erg (energy)	g cm^2 s^{-2}	
n = 10^{-9}	k = 10^3	M = 10^6
μ = 10^{-6}	m = 10^{-3}	G = 10^9