

### Electromagnetic Radiation

$$c = \lambda\nu \quad E = h\nu = \frac{hc}{\lambda}$$

#### Planck Function

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/k_B T} - 1}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda k_B T} - 1}$$

#### Wien's Displacement Law

$$\lambda_{\text{peak}} T = 2.898 \times 10^6 \text{ nm} \cdot \text{K}$$

#### Stefan-Boltzmann

$$F_\star = \sigma T_{\text{eff}}^4 \quad L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

#### Planetary Equilibrium Temp ( $A = \text{albedo}$ )

$$T_{\text{eq}} = \left( \frac{L_\star(1-A)}{16\pi\sigma d^2} \right)^{1/4}$$

### Brightness, Magnitudes, Distance

$$F = \frac{L}{4\pi d^2}$$

(apparent flux)

$$m_1 - m_2 = -2.5 \log_{10}(F_1/F_2)$$

$$m - M = 5 \log_{10}(d/10\text{pc})$$

$$M = 4.83 - 2.5 \log_{10}(L/L_\odot)$$

$$d[\text{pc}] = 1/p[\text{arcsec}]$$

$$1 \text{ rad} = 206,265 \text{ arcsec}$$

### Kirchhoff's Spectroscopy Laws

(1) Hot dense source → continuous. (2) Hot low-density gas → emission lines. (3) Cool gas before hot source → absorption lines.

### Gravity, Energy, Virial

$$F_g = \frac{Gm_1 m_2}{r^2} \quad g = \frac{GM}{R^2}$$

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}} \quad v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

$$E_K = \frac{1}{2}mv^2 \quad U = -\frac{GMm}{r}$$

$$E = E_K + U \quad (E < 0 \text{ bound})$$

#### Virial (bound):

$$2E_K + U = 0$$

#### Binding-energy scale:

$$E_g \sim -GM^2/R$$

#### Bound orbit:

$$E = -GMm/(2a)$$

### Kepler & Binary Orbits

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

$$\frac{M_1 + M_2}{M_\odot} = \frac{(a/\text{AU})^3}{(P/\text{yr})^2}$$

#### Spec. binary:

$$K_1/K_2 = M_2/M_1$$

Center of mass:  $M_1 a_1 = M_2 a_2$ ,  $a = a_1 + a_2$ .

#### Main-Sequence Scalings

$$L \propto M^{3.5} \quad t_{\text{MS}} \propto M/L \propto M^{-2.5}$$

### Doppler & Telescopes (non-rel., $v \ll c$ )

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

$$A_{\text{coll}} = \pi D^2/4 \quad \alpha_\lambda = 1.22\lambda/D \quad R = \lambda/\Delta\lambda$$

### Stellar Structure Equations

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

(mass continuity)

$$\frac{dP}{dr} = -\rho \frac{GM(r)}{r^2}$$

(HSE)

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

(energy gen.)

$$\frac{dT}{dr} = -\frac{3\kappa\rho L(r)}{16\pi acT^3 r^2}$$

(rad. transp.)

#### One-Zone Scalings

$$P_c \sim \frac{GM^2}{R^4} \quad \rho_c \sim M/R^3$$

$$T_c \sim \frac{\mu GM m_p}{k_B R} \quad g_{\text{surf}} \sim (GM)/R^2$$

$$E_g \sim -(GM^2)/R$$

#### Toy MS scalings (radiative stars, from structure eqs.):

$$L \propto M^3/\kappa \quad R \propto M^{3/7}$$

### Pressure Laws

$$P_{\text{gas}} = nk_B T = \frac{\rho k_B T}{\mu m_p}$$

$$P_{\text{rad}} = \left(\frac{1}{3}\right) a T^4 \quad (a = 4\sigma/c)$$

$$P_{\text{deg, NR}} \propto \rho^{5/3}/m_e \quad P_{\text{deg, R}} \propto \rho^{4/3}$$

$$P_{\text{tot}} = P_{\text{gas}} + P_{\text{rad}} + P_{\text{deg}}$$

#### Mean mol. weight (ionized):

$$1/\mu \approx 2X + (3/4)Y + (1/2)Z$$

solar  $\mu \approx 0.6$ , pure-H  $\mu \approx 0.5$ .

### Stellar Timescales

$$t_{\text{dyn}} \sim \sqrt{R^3/GM} \sim 1/\sqrt{G\rho}$$

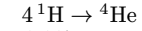
$$t_{\text{KH}} \sim |E_g|/L \sim \frac{GM^2}{RL}$$

$$t_{\text{nuc}} \sim \frac{\epsilon_{\text{nuc}} f M c^2}{L}$$

$f \sim 0.1$  (fusible mass fraction for H)  $\epsilon_{\text{nuc}} \approx 0.007$  (H-fusion eff.)

### Nuclear Fusion

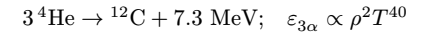
#### H fusion:



,  $\Delta E \approx 26.7 \text{ MeV}$ ,  $\epsilon_{\text{H}} \approx 0.007$ .

$$\epsilon_{\text{pp}} \propto \rho T^4 \quad \epsilon_{\text{CNO}} \propto \rho T^{17}$$

#### Triple- $\alpha$ :



#### Coulomb Barrier & Tunneling

$$U_{\text{Coul}}(r) = \frac{Z_1 Z_2 e^2}{r} \quad \lambda_{\text{dB}} = h/p$$

$$\Gamma \approx e^{-2G}; \quad G \propto (Z_1 Z_2)/\sqrt{T}$$

(Gamow)

### Radiation Transport

$$\ell = 1/(\kappa\rho) \quad d\tau = \kappa\rho dr$$

$$t_{\text{diff}} \sim R^2/(\ell c) = (\kappa\rho R^2)/c$$

$\kappa_{\text{es}} = 0.34 \text{ cm}^2 \text{ g}^{-1}$  (H-rich).

#### Eddington Luminosity

$$L_{\text{Edd}} = \frac{4\pi GMc}{\kappa} \quad L_{\text{Edd}, \odot} \approx 3.9 \times 10^4 L_\odot$$

### Quantum Mechanics

$$\Delta p \Delta x \gtrsim \hbar/2 \quad \lambda_{\text{dB}} = h/p$$

$$\hbar = h/(2\pi)$$

### White Dwarfs

$$R_{\text{WD}} \propto M^{-1/3} \quad R_{\text{WD}} \approx (0.01 R_\odot) (M_{\text{WD}}/0.7 M_\odot)^{-1/3}$$

$$R_{\text{WD}} \sim 10^{-2} R_\odot \sim R_\delta$$

$$M_{\text{Ch}} \approx 1.4 M_\odot$$

(theoretical upper mass limit for stable WD)

### Stellar Mass Limits

$$M_{\text{min}} \approx 0.08 M_\odot \quad M_{\text{max}} \sim 100\text{--}150 M_\odot$$

### Math Reference

**Order of Operations — PEMDAS** Parentheses → Exponents → Multiplication / Division (L → R) → Addition / Subtraction (L → R)

#### Exponents

$$y^a y^b = y^{a+b}; (y^a)^b = y^{ab}; y^{-a} = 1/y^a; y^{1/n} = \sqrt[n]{y}$$

#### Logarithms

$$\log(ab) = \log a + \log b; \log(a/b) = \log a - \log b$$

$$\log(a^b) = b \log a; \log_{10} 10^a = a; \ln e^a = a$$

#### Sphere Geometry

$$V = \frac{4}{3}\pi r^3 \quad A_s = 4\pi r^2 \quad A_p = \pi r^2 \quad C = 2\pi r$$

#### Useful Approximations (small $x$ )

$$\sin \theta \approx \theta \quad (1+x)^n \approx 1+nx \quad e^x \approx 1+x$$

### Ratio Method

For  $Q \propto A^a B^b$ :

$$Q_2/Q_1 = (A_2/A_1)^a (B_2/B_1)^b$$

Use **before** plugging in numbers — units and prefactors cancel.

### Astronomical Values & Conversions

$M_\odot$	$2.0 \times 10^{33}$ g
$R_\odot$	$7.0 \times 10^{10}$ cm
$L_\odot$	$3.8 \times 10^{33}$ erg s <sup>-1</sup>
$T_{\text{eff},\odot}$	5800 K
$T_{c,\odot}$	$1.5 \times 10^7$ K
$\rho_{c,\odot}$	$150$ g cm <sup>-3</sup>
$t_{\text{MS},\odot}$	10 Gyr
$M_{\text{Jup}}, R_{\text{Jup}}$	$1.9 \times 10^{30}$ g, $7.0 \times 10^9$ cm
$M_\delta, R_\delta$	$6.0 \times 10^{27}$ g, $6.4 \times 10^8$ cm
1 pc	$3.086 \times 10^{18}$ cm = 3.26 ly
1 ly	$9.46 \times 10^{17}$ cm
1 AU	$1.496 \times 10^{13}$ cm
1 yr	$3.156 \times 10^7$ s

### Physical Constants (CGS)

Quantity	Value
$c$	$3.0 \times 10^{10}$ cm s <sup>-1</sup>
$G$	$6.67 \times 10^{-8}$ cm <sup>3</sup> g <sup>-1</sup> s <sup>-2</sup>
$k_B$	$1.38 \times 10^{-16}$ erg K <sup>-1</sup>
$\sigma$	$5.67 \times 10^{-5}$ erg cm <sup>-2</sup> s <sup>-1</sup> K <sup>-4</sup>
$a = 4\sigma/c$	$7.57 \times 10^{-15}$ erg cm <sup>-3</sup> K <sup>-4</sup>
$h$	$6.63 \times 10^{-27}$ erg s
$\hbar$	$1.05 \times 10^{-27}$ erg s
$m_p$	$1.67 \times 10^{-24}$ g
$m_n$	$1.68 \times 10^{-24}$ g
$m_e$	$9.11 \times 10^{-28}$ g
amu	$1.66 \times 10^{-24}$ g
$e$	$4.80 \times 10^{-10}$ esu
1 eV	$1.60 \times 10^{-12}$ erg
$hc$	1240 eV nm
$\kappa_{\text{es}}$	$0.34$ cm <sup>2</sup> g <sup>-1</sup>

### CGS Derived Units

Quantity	Unit	Base units
Force	dyne	g cm s <sup>-2</sup>
Energy	erg	g cm <sup>2</sup> s <sup>-2</sup>
Power	erg s <sup>-1</sup>	g cm <sup>2</sup> s <sup>-3</sup>
Pressure	dyne cm <sup>-2</sup>	g cm <sup>-1</sup> s <sup>-2</sup>
Density	g cm <sup>-3</sup>	—
Charge	esu	g <sup>1/2</sup> cm <sup>3/2</sup> s <sup>-1</sup>

### Powers-of-Ten Prefixes

n (nano)	10 <sup>-9</sup>	k (kilo)	10 <sup>3</sup>
$\mu$ (micro)	10 <sup>-6</sup>	M (mega)	10 <sup>6</sup>
m (milli)	10 <sup>-3</sup>	G (giga)	10 <sup>9</sup>
c (centi)	10 <sup>-2</sup>	T (tera)	10 <sup>12</sup>