# Homework 2

## Gravity & Orbits II + Light as Information

Dr. Anna Rosen

## Learning Objectives

- Interpret the **sign of orbital energy** and connect it to bound vs. unbound motion.
- Use **Newton + Kepler** to move between periods, masses, and orbital sizes.
- Apply **escape velocity** and **orbital velocity** as scaling tools with unit checks.
- Use the **virial theorem** to connect motions to binding energy.
- Use **spectra and Doppler shifts** to infer velocities and physical properties.
- Treat equations as **stories with units**, not just recipes.

## Concept Throughline

- **Measurement → model → inference** remains the core chain.
- **Energy sets orbit type** (bound, parabolic, unbound).
- **Light is information**: spectra and Doppler shifts let us infer motion and composition.
- **Scaling beats big numbers** when the physics is clear.

## Prerequisites

- Scientific notation and exponent rules
- Basic algebra and proportional reasoning
- Unit conversion (cm, km, s; g, kg; nm)

## Relevant Readings

- Lecture 3: Gravity and Orbits
- Lecture 4: Light as Information

> **i** Note
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> **Before you start:** Review the Homework Guidelines for required format and tools.

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## Problems (10 total)

### Part A — Gravity & Orbits

### Problem 1 — Energy Signs and Orbital Fate

The total mechanical energy is $E = \frac{1}{2}mv^2 - GMm/r$.

- (a) Why is gravitational potential energy negative (with zero at infinity)?
- (b) What does $E < 0$, $E = 0$, and $E > 0$ mean physically?
- (c) A comet approaches the Sun with $E > 0$. A student says, "It has positive energy, so it's gaining energy as it approaches." Correct this.

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### Problem 2 — Orbital Velocity Practice

Using $v_{orb} = \sqrt{GM/r}$:

- (a) Calculate Earth's orbital velocity around the Sun using $M_\odot = 2 \times 10^{33}$ g and $r = 1$ AU $= 1.5 \times 10^{13}$ cm. Express in cm/s and km/s.
- (b) Show explicitly that $\sqrt{GM/r}$ has units of velocity.
- (c) Mars orbits at 1.52 AU. Using ratios, find Mars's orbital velocity.
- (d) Sanity check: should Mars move faster or slower than Earth?

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### Problem 3 — Escape Velocity Comparison

Using $v_{esc} = \sqrt{2GM/R}$:

- (a) Calculate Earth's escape velocity. Use $M_\oplus = 6 \times 10^{27}$ g and $R_\oplus = 6.4 \times 10^8$ cm.
- (b) The Moon has $M_{Moon} = 7.3 \times 10^{25}$ g and $R_{Moon} = 1.7 \times 10^8$ cm. Calculate its escape velocity.
- (c) By what factor is Earth's escape velocity larger than the Moon's?
- (d) Interpret: why did Apollo astronauts need a smaller rocket to leave the Moon?

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**Problem 4 — Kepler III with Newton**

Newton's version is $P^2 = \dfrac{4\pi^2}{G(M+m)}a^3$.

- (a) For Earth, $m \ll M_\odot$. Show the simplified form.
- (b) Verify it gives $P \approx 1$ year for $a = 1$ AU.
- (c) Confirm both sides have units of s$^2$.
- (d) A binary star has two equal $1\,M_\odot$ stars. How does its Kepler constant compare to the Sun-Earth system?

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**Problem 5 — Complete Workflow: Weighing Jupiter**

Io orbits Jupiter with period $P = 1.77$ days and semi-major axis $a = 4.22 \times 10^{10}$ cm.

- (a) Use Newton's Kepler III to solve for Jupiter's mass.
- (b) Unit check: verify $\frac{a^3}{GP^2}$ has units of mass.
- (c) Express your answer in grams and in solar masses.
- (d) What observable quantities did you need?

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**Problem 6 — The $\sqrt{2}$ Factor**

Escape velocity is $v_{esc} = \sqrt{2}\,v_{orb}$ at any radius.

- (a) Derive this relationship from $v_{orb} = \sqrt{GM/r}$ and $v_{esc} = \sqrt{2GM/r}$.
- (b) A spacecraft in circular orbit wants to escape. By what factor must it increase its speed?
- (c) Explain the energy interpretation: why does escape require adding kinetic energy equal to the orbital kinetic energy?

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**Problem 7 — Virial Theorem Capstone**

A star cluster has total kinetic energy $K = 1.0 \times 10^{50}$ erg in stellar motions and is in virial equilibrium $(2K + U = 0)$.

- (a) Use the virial theorem to find its gravitational potential energy $U$.
- (b) Compute the total energy $E = K + U$ and state whether the cluster is bound.
- (c) A tidal interaction injects an additional $0.6 \times 10^{50}$ erg of kinetic energy without changing the cluster size. What happens to the sign of $E$? Does the cluster remain bound?

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**Part B — Light as Information**

**Problem 8 — Kirchhoff's Laws in Action**

You see three spectra from three different sources:

- (a) A smooth rainbow with dark lines
- (b) A smooth rainbow with no lines
- (c) A dark background with bright lines

For each, identify the physical source (hot dense object, hot thin gas, or cool gas in front of a hot source). Explain your reasoning.

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**Problem 9 — Why Skies Are Red at Sunset**

Use Rayleigh scattering to explain why the Sun looks red near the horizon but not at noon. What changes about the path through the atmosphere, and how does $\lambda^{-4}$ make the effect strong?

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**Problem 10 — Photon Energy from Wavelength**

An orange photon has $\lambda = 600$ nm.

- (a) Convert this wavelength to cm.
- (b) Compute its frequency.
- (c) Compute its energy in erg using $E = hc/\lambda$ with $h = 6.626 \times 10^{-27}$ erg $\cdot$ s.