Homework 3

Parallax + Surface Flux & Radiation Inference

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Learning Objectives

- Use **parallax geometry** to infer stellar distance and avoid common interpretation errors.
- Apply the **inverse-square law** to connect observed flux, luminosity, and distance.
- Propagate uncertainty along the chain: **parallax** → **distance** → **luminosity**.
- Use **Wien's law** to infer effective temperature from peak wavelength/color.
- Use **Stefan-Boltzmann scaling** to infer surface flux and stellar radius.
- Synthesize the full **observable** → **model** → **inference** chain for a star.

Concept Throughline

- Measured angular shift gives distance.
- Distance plus measured flux gives intrinsic luminosity.
- Color gives temperature.
- Luminosity plus temperature gives radius.
- Every step has assumptions; good inference means checking them explicitly.

Prerequisites

- Scientific notation and proportional reasoning
- Unit conversion (arcsec, pc, cm, nm, K)
- Module 1 light basics (Wien + Planck ideas)

Relevant Sources (Module-Based)

- Module 2 (slides): Distance & Parallax
- Module 2 (reading/source set): Surface Flux & Colors of Stars
- Module 1 review: Light as Information

> **Note**
>
> **Before you start:** Review the Homework Guidelines for required format and tools.

Tip

HW3 note: Tool hints are not shown. Part of the goal is deciding which model and scaling apply.
Use ratios whenever possible to avoid unnecessary constants.

Note

Use these constants unless a problem states otherwise:

- 1 pc = 206,265 AU
- 1 AU = 1.496×10^{13} cm
- 1 pc = 3.086×10^{18} cm
- $T_\odot = 5800$ K
- $b = 2.898 \times 10^6$ nm · K

Note

Required reporting format:

- Every numeric answer must include units.
- Use scientific notation where appropriate.
- Include a one-line sanity check for each problem (e.g., farther should look dimmer; hotter should peak at shorter wavelength).

Tip

Sanity-check scalings:

- $d(\text{pc}) = 1/p('')$
- $F \propto L d^{-2}$
- $T = b/\lambda_{\text{peak}}$
- $L \propto R^2 T^4$

Problems (10 total)

Part A — Distance, Parallax, and Luminosity

Problem 1 — Parallax Is Not Angular Size

A student says: "This nearby star has a large parallax, so it must have a large angular size."

- Be explicit: parallax is an **apparent positional shift against distant background**, not an apparent size.

- (a) Explain why this statement is incorrect.
- (b) Define parallax angle and angular size in one sentence each.
- (c) Give one observational signature that distinguishes the two measurements.

Problem 2 — Parallax to Distance

- Do the factor comparison in pc first; convert to cm afterward.
- (a) Star A has $p = 0.50''$. Find d in pc and in cm.
- (b) Star B has $p = 0.020''$. Find d in pc and in cm.
- (c) Which star is farther, and by what factor?

Problem 3 — Inverse-Square Flux Scaling

Two identical stars have the same luminosity. Star X is at 20 pc and Star Y is at 50 pc. Assume no extinction/reddening.

- (a) Compute F_X/F_Y using inverse-square scaling.
- (b) Which star appears brighter at Earth?
- (c) In one sentence, explain why equal luminosity does not imply equal observed flux.

Problem 4 — Luminosity from Flux + Distance

A star has measured parallax $p = 0.10''$ and bolometric flux at Earth $F_\star = 2.0 \times 10^{-11}\, F_\odot$, where $F_\odot$ is specifically the Sun's bolometric flux measured at **1 AU**.

- (a) Compute distance in pc and in AU.
- (b) Use

$$\frac{L_\star}{L_\odot} = \frac{F_\star}{F_\odot}\left(\frac{d_\star}{1\,\mathrm{AU}}\right)^2$$

  since $F_\odot$ is measured at 1 AU.
- (c) State whether the star is more or less luminous than the Sun, and by what factor.

Problem 5 — Same Flux, Different Parallax

Two stars have the same measured bolometric flux at Earth. Their parallaxes are:

- Star A: $p_A = 0.100''$
- Star B: $p_B = 0.025''$
- (a) Which star is farther, and by what factor?
- (b) Infer L_B/L_A using a ratio method.
- (c) In 2–3 sentences, explain why equal observed flux does not imply equal intrinsic luminosity.
- (d) Include a one-line equation showing the scaling you used at fixed observed flux.

Part B — Radiation, Temperature, and Radius

Problem 6 — Same Luminosity, Different Color

Two stars have the same luminosity. One appears blue, the other red.

- (a) Which star is hotter?
- (b) Which star has the larger radius?
- (c) Justify your answer using Stefan-Boltzmann scaling, not just verbal intuition.

Problem 7 — Temperature from Wien's Law

A star's spectrum peaks at $\lambda_{\text{peak}} = 725$ nm.

- (a) Compute the star's effective temperature from the peak wavelength (report to 2 significant figures).
- (b) Compare this temperature to the Sun's by computing $T/T_{\odot}$.
- (c) State whether this star is hotter or cooler than the Sun.

Problem 8 — Planck Limits and Physical Meaning

Goal: show how Planck's law reduces to Rayleigh-Jeans behavior at long wavelength and becomes exponentially suppressed at short wavelength.

Consider the Planck function

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1}.$$

- (a) Define $x = hc/(\lambda k_B T)$. In the long-wavelength limit ($x \ll 1$), use an appropriate approximation to show the leading scaling of B_λ with T and λ. (Hint: $e^x \approx 1 + x$ for $x \ll 1$.)
- (b) In the short-wavelength limit ($x \gg 1$), show the leading scaling behavior of B_λ and identify which term suppresses emission. (Hint: $e^x - 1 \approx e^x$ for $x \gg 1$.)
- (c) Explain in words why these two limits resolve the ultraviolet-catastrophe issue from classical physics.
- (d) Connect your result to the statement: "photons are expensive when $h\nu \gg k_B T$."

Problem 9 — Radius from Luminosity and Temperature

A star has luminosity $L = 16\,L_\odot$ and temperature $T = 2\,T_\odot$.

- (a) Solve for $(R/R_\odot)^2$.
- (b) Solve for $R/R_\odot$.
- (c) Interpret whether the star is physically larger or smaller than the Sun.

Problem 10 — Full Chain: Parallax + Flux + Color

You observe a star with:

- Parallax: $p = 0.050''$
- Bolometric flux at Earth: $F_\star = 1.5 \times 10^{-12}\,F_\odot$
- Peak wavelength: $\lambda_{\text{peak}} = 500$ nm
- (a) Compute distance in pc and in AU.
- (b) Infer luminosity in units of $L_\odot$ using a ratio method.
- (c) Compute temperature from the peak wavelength.
- (d) Compute radius in solar units using the appropriate radiation relation.
- (e) Assume the parallax is measured as $p = 0.050'' \pm 0.003''$. Estimate the fractional uncertainty in distance, and then in luminosity (assuming flux uncertainty is negligible). (Use fractional uncertainty: if $d \propto p^{-1}$, then $\delta d/d \approx \delta p/p$.)

- (f) In 2–4 sentences, summarize the full observable $\rightarrow$ model $\rightarrow$ inference chain you used.