Homework 4

Surface Flux & Colors (continued) + Spectra & Composition

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Learning Objectives

- Use **Stefan-Boltzmann scaling** ($L \propto R^2 T^4$) to connect luminosity, temperature, and radius via ratios.
- Interpret how **interstellar extinction** biases temperature and radius inferences.
- Locate stars on the **HR diagram** and connect position to physical size.
- Apply **Kirchhoff's laws** to identify spectrum types from physical conditions.
- Use the **Bohr model** and energy-level ratios to predict hydrogen line wavelengths.
- Apply the **Doppler formula** ($\Delta\lambda/\lambda_0 = v_r/c$) to infer radial velocity from spectral-line shifts.
- Calculate **planetary equilibrium temperatures** using the ratio method and interpret greenhouse warming.

Concept Throughline

- Temperature and luminosity together determine radius — Stefan-Boltzmann as a diagnostic.
- Spectral lines encode composition, temperature, and velocity — three inferences from one observation.
- The same quantum physics (energy levels + absorption) governs stellar atmospheres and planetary climates.
- Ratio methods and scaling arguments are more powerful than absolute calculations.

Prerequisites

- Scientific notation and proportional reasoning
- Stefan-Boltzmann and Wien's law from Module 2, Lecture 2
- Module 1 light basics (spectra, blackbody ideas)

Relevant Sources (Module-Based)

- Module 2 (reading): Surface Flux & Colors of Stars
- Module 2 (reading): Spectra & Composition

Note

Before you start: Review the Homework Guidelines for required format and tools.

Tip

HW4 note: This assignment emphasizes **ratio and scaling methods**. Use solar units and dimensionless ratios whenever possible. Avoid absolute CGS calculations unless a problem explicitly requires them.
Use ratios whenever possible to avoid unnecessary constants.

Note

Use these constants unless a problem states otherwise:

- $T_\odot = 5800$ K
- $R_\odot = 6.96 \times 10^{10}$ cm
- $L_\odot = 3.828 \times 10^{33}$ erg/s
- $b = 2.898 \times 10^6$ nm · K
- $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
- E_n(hydrogen) $= -13.6$ eV$/n^2$
- $c = 3.0 \times 10^5$ km/s
- Hα rest wavelength: $\lambda_0 = 656.3$ nm
- Na D rest wavelength: $\lambda_0 = 589.0$ nm
- $T_{\rm ref} = 279$ K (equilibrium temperature of a zero-albedo planet at 1 AU from the Sun)

Note

Required reporting format:

- Every numeric answer must include units.
- Use scientific notation where appropriate.
- Include a one-line sanity check for each problem (e.g., cooler star → larger radius at fixed L; redshifted → receding).

Tip

Sanity-check scalings:

- $L \propto R^2 T^4$ (Stefan-Boltzmann)
- $T = b/\lambda_{\rm peak}$ (Wien's law)
- $R \propto T^{-2}$ at fixed L
- $\Delta\lambda/\lambda_0 = v_r/c$ (Doppler)
- $T_{\rm eq} \propto (1 - A)^{1/4}\, d^{-1/2}$ (planetary equilibrium)

Problems (10 total)

Part A — Surface Flux, Radius, and the HR Diagram

Problem 1 — Why Betelgeuse Is Huge

Betelgeuse has:

- Luminosity: $L \sim 10^5 \, L_\odot$ (100,000 times the Sun)
- Temperature: $T \sim 3{,}500$ K (about 60% of the Sun's temperature)

Use Stefan-Boltzmann to explain qualitatively why Betelgeuse must have an enormous radius despite being cooler than the Sun. Your answer should reference the surface flux $F_\star = \sigma T^4$ (power per unit area) and the surface area $4\pi R^2$.

Problem 2 — Rigel's Radius (Ratio Method)

Rigel (a blue supergiant) has:

- Luminosity: $L_{\text{Rigel}} = 120{,}000 \, L_\odot$ (inferred from parallax + received flux)
- Effective temperature: $T_{\text{Rigel}} = 12{,}100$ K (inferred from color via Wien's law)

Using the solar-unit form of Stefan-Boltzmann:

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$$

- (a) Compute the temperature ratio $T_{\text{Rigel}}/T_\odot$ and verify it is dimensionless.
- (b) Raise to the fourth power: $(T_{\text{Rigel}}/T_\odot)^4 = ?$
- (c) Compute $(R_{\text{Rigel}}/R_\odot)^2$, then take the square root to find $R_{\text{Rigel}}/R_\odot$.
- (d) Is Rigel larger or smaller than the Sun? Betelgeuse has $R_{\text{Betelgeuse}} \approx 870 \, R_\odot$. How does Rigel compare? Both are supergiants — explain in 1–2 sentences why one is so much larger despite both being extremely luminous.

Problem 3 — HR Diagram Reasoning: Three Stars

Three stars lie on the HR diagram:

- Star A: $T = 6{,}000$ K, $L = 1 \, L_\odot$ (like the Sun)
- Star B: $T = 3{,}000$ K, $L = 100 \, L_\odot$ (red giant)
- Star C: $T = 12{,}000$ K, $L = 1 \, L_\odot$ (hot but dim)

- (a) Using Stefan-Boltzmann in solar units, calculate the radius of each star in solar radii. For each star, show $(R/R_{\odot})^2$ first, then take the square root.
- (b) Which star is largest? Which is smallest?
- (c) Sketch these three points on an HR diagram (luminosity vs. temperature, with T increasing to the left). Draw approximate lines of constant radius through each point. What do these lines tell you about the giant and white-dwarf regions of the HR diagram?

Problem 4 — The Effect of Extinction on Temperature Inference

Interstellar dust reddens starlight by scattering blue photons more than red ones. An observer measures a star and finds:

- Observed peak wavelength: $\lambda_{\text{obs}} = 700$ nm (appears very red)
- True peak wavelength (after correcting for dust): $\lambda_{\text{true}} = 500$ nm
- (a) Calculate the apparent temperature from the observed (reddened) color using Wien's law. Show units.
- (b) Calculate the true temperature from the corrected color. Show units.
- (c) By what factor does dust extinction bias the inferred temperature? Compute $T_{\text{apparent}}/T_{\text{true}}$.
- (d) If an astronomer used the reddened temperature to infer the star's radius via Stefan-Boltzmann (at the correct luminosity), would the inferred radius be too large or too small? By what factor? (Hint: $R \propto T^{-2}$ at fixed L.)

Part B — Spectra, Doppler, and Planetary Climate

Problem 5 — The OBAFGKM Misconception

A student says: "O stars show strong helium lines because they have more helium than cooler stars."

Explain why this is wrong. What actually controls the strength of helium lines? Your answer should reference quantum energy levels and the temperature needed to populate the relevant atomic states.

Problem 6 — Doppler Direction

Two stars are observed with Hα lines at:

- Star X: $\lambda_{\text{obs}} = 656.8$ nm
- Star Y: $\lambda_{\text{obs}} = 655.8$ nm

The Hα rest wavelength is $\lambda_0 = 656.3$ nm.

- (a) Which star is approaching and which is receding?
- (b) Without calculating, which star has the larger radial velocity magnitude? Explain your reasoning.
- (c) Verify your answer to (b) by computing both radial velocities using $v_r = c \times \Delta\lambda/\lambda_0$.

Problem 7 — Hydrogen Line Ratios from the Bohr Model

The Bohr model gives hydrogen energy levels $E_n = -13.6$ eV$/n^2$. The Balmer series consists of transitions from upper levels $n > 2$ down to $n = 2$. Hα ($n = 3 \to 2$) is observed at $\lambda_{H\alpha} = 656.3$ nm.

- (a) Write the energy of the $n = 2$, $n = 3$, and $n = 4$ levels as fractions of 13.6 eV (e.g., $E_2 = -13.6$ eV$/4 = -3.40$ eV). No unit conversions needed — stay in eV throughout.
- (b) Compute the energy differences $\Delta E_{H\alpha} = E_3 - E_2$ and $\Delta E_{H\beta} = E_4 - E_2$, both in eV. Then compute the dimensionless ratio $\Delta E_{H\beta}/\Delta E_{H\alpha}$.
- (c) Since $\lambda = hc/\Delta E$, we have $\lambda_{H\beta}/\lambda_{H\alpha} = \Delta E_{H\alpha}/\Delta E_{H\beta}$ (the ratio inverts). Use this plus $\lambda_{H\alpha} = 656.3$ nm to predict $\lambda_{H\beta}$ in nm. Compare to the observed value of 486.1 nm.
- (d) Without computing exact values, will Hγ ($n = 5 \to 2$) have a longer or shorter wavelength than Hβ? As $n \to \infty$, what wavelength does the Balmer series approach? (Hint: what is ΔE for $n = \infty \to 2$ in eV, and what does the wavelength ratio $\lambda_{\text{limit}}/\lambda_{H\alpha}$ give?)

Problem 8 — Doppler Velocity

A star's sodium D line (rest wavelength $\lambda_0 = 589.0$ nm) is observed at $\lambda_{\text{obs}} = 589.4$ nm.

- (a) Is the star approaching or receding?
- (b) Calculate the radial velocity in km/s using $v_r = c \times \Delta\lambda/\lambda_0$.
- (c) Express the fractional shift $\Delta\lambda/\lambda_0$ as a percentage — how does it compare to the ratio v_r/c?

Problem 9 — Planetary Equilibrium Temperature (Ratio Method)

Calculate the equilibrium temperature of a planet at $d = 1.5$ AU from a Sun-like star ($L_\star = L_\odot$), with albedo $A = 0.25$.

- (a) Use the ratio method from the reading:

$$T_{\rm eq} = 279\ \text{K} \times \left(\frac{L_\star}{L_\odot}\right)^{1/4} \times (1 - A)^{1/4} \times \left(\frac{d}{1\ \text{AU}}\right)^{-1/2}$$

  Evaluate each factor separately, then multiply. Show units at each step.
- (b) Compare your result to Mars's actual surface temperature (~ 210 K). Are they consistent?
- (c) What does the comparison in (b) tell you about the strength of Mars's greenhouse effect?

Problem 10 — Venus's Runaway Greenhouse

Venus has $d = 0.72$ AU from the Sun, albedo $A = 0.77$ (highly reflective sulfuric acid clouds), and an atmosphere that is 96.5% CO_2 at 92 bar surface pressure.

- (a) Calculate Venus's equilibrium temperature using the ratio method. Show each factor separately.
- (b) Compare to the actual surface temperature of 735 K. Calculate the greenhouse warming $\Delta T = T_{\rm surface} - T_{\rm eq}$.
- (c) The greenhouse warming on Venus ($\Delta T \approx 508$ K) is about 15× larger than on Earth ($\Delta T \approx 33$ K). Using the molecular absorption concept from the reading, explain qualitatively why a thicker CO_2 atmosphere produces a much larger greenhouse effect.
- (d) Venus is closer to the Sun than Earth, yet its equilibrium temperature is *lower* than Earth's ($T_{\rm eq,Venus} < T_{\rm eq,Earth}$). Explain how this is possible. (Hint: look at the albedo term.)
- (e) Could Earth experience a runaway greenhouse? In 2–3 sentences, describe what physical conditions would need to change.