Homework 5

Masses from Motion + Magnitudes/HR Synthesis (Midterm 1 Review)

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Learning Objectives

- Use binary-star dynamics to infer stellar masses from observables.
- Connect radial velocity amplitudes to mass ratio and interpret what is directly measured versus inferred.
- Apply the magnitude system and distance modulus to move between apparent brightness, absolute brightness, and distance.
- Use ratio-based stellar scalings to connect mass, luminosity, and main-sequence lifetime.
- Synthesize multiple observables into a coherent observable → model → inference chain.

Concept Throughline

- **You do not measure stellar mass directly.** You infer it from motion.
- **Brightness is not luminosity.** Distance and logarithmic magnitudes matter.
- **The HR diagram is a map of physics, not just a plot.**
- **Scaling + sanity checks** should guide every quantitative step.

Prerequisites

- Module 1 tools: unit handling, scaling, interpretation checks
- Kepler/Newton orbital reasoning from Module 1 and Module 2
- Doppler/radial velocity basics
- Logarithms and scientific notation

Relevant Sources (Module-Based)

- Module 2 (reading): The Last Piece — Weighing Stars
- Module 2 (reading): The HR Diagram — Finding Patterns, Needing Models
- Module 1 review: Gravity and Orbits
- Module 1 review: Light as Information

Note

Before you start: Review the Homework Guidelines for required format and tools.

Tip

HW5 note: This is your last pre-midterm assignment. Tool hints are not shown. Choose methods deliberately and include one-line sanity checks.

Note

Use these constants and relations unless a problem states otherwise:

- Solar-unit Kepler form (for binary total mass):

$$\frac{M_1 + M_2}{M_\odot} = \frac{(a/\mathrm{AU})^3}{(P/\mathrm{yr})^2}$$

- SB2 radial-velocity amplitude relation:

$$\frac{M_2}{M_1} = \frac{K_1}{K_2}$$

  Reminder: the more massive star has the smaller radial-velocity amplitude (K).
- Distance modulus:

$$m - M = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)$$

- Main-sequence scaling (approx.):

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{3.5}$$

- Lifetime scaling (approx.):

$$t_{\mathrm{MS}} \propto \frac{M}{L} \propto M^{-2.5}, \quad t_\odot \approx 10\,\mathrm{Gyr}$$

Note

Required reporting format:

- Every numeric answer must include units.
- Use scientific notation where appropriate.
- Include a one-line sanity check for each problem.

Tip

Sanity-check scalings:

- $M_{\text{tot}} \propto a^3/P^2$
- $K_1/K_2 = M_2/M_1$
- $m - M$ increases with distance
- $L \propto M^{3.5}$ and $t_{\text{MS}} \propto M^{-2.5}$

Problems (10 total)

Part A — Masses from Motion + Magnitudes

Problem 1 — Why "Weighing a Star" Is an Inference

A student says: "Astronomers can't really know stellar masses because nobody can put a star on a scale."

- (a) Identify one direct observable used to infer stellar mass in binaries.
- (b) Name the physical model that turns that observable into mass.
- (c) In 2–4 sentences, explain why this is still a valid measurement in scientific practice.

Problem 2 — Total Mass from a Visual Binary

A visual binary has orbital period $P = 8.0\,\text{yr}$ and semi-major axis $a = 4.0\,\text{AU}$.

- (a) Compute the total system mass $(M_1 + M_2)$ in solar masses.
- (b) If the two stars have equal mass, what is each mass?
- (c) Is this system total mass larger or smaller than the Sun's mass? Give a one-line interpretation.

Problem 3 — Mass Ratio from SB2 Velocities

For a double-lined spectroscopic binary (SB2), you measure radial-velocity amplitudes:

- $K_1 = 30\,\mathrm{km/s}$
- $K_2 = 45\,\mathrm{km/s}$

The total mass is known from independent orbital data to be $M_1 + M_2 = 3.0\,M_\odot$.

- (a) Compute the mass ratio M_2/M_1.
- (b) Solve for M_1 and M_2 in solar masses.
- (c) Which star is more massive, and how can you tell from the velocity amplitudes?

Problem 4 — Inclination and the Hidden-Mass Problem

Two binaries have identical measured periods and identical radial-velocity amplitudes, but one system is eclipsing and the other is not.

- (a) Why does the eclipsing system usually allow a better mass determination?
- (b) What is the role of orbital inclination in turning observed radial velocity into true orbital speed?
- (c) Explain why a nearly face-on system can hide large true masses.

Problem 5 — Required Capstone: Eclipsing SB2 Full Inference Chain

> **Note**
>
> **Required capstone:** This is the longest problem on HW5 and is intentionally integrative.

An eclipsing SB2 system has:

- Period: $P = 2.00\,\mathrm{yr}$
- Semi-major axis: $a = 2.00\,\mathrm{AU}$
- Velocity amplitudes: $K_A = 40\,\mathrm{km/s}$ and $K_B = 20\,\mathrm{km/s}$

Assume both stars are main-sequence and use $L/L_\odot \approx (M/M_\odot)^{3.5}$. Assume a is the true semi-major axis (not a projected/angular separation).

- (a) Compute $M_A + M_B$.
- (b) Compute the mass ratio and then M_A and M_B.
- (c) Which star is more massive? Which star moves faster in orbit?
- (d) Estimate the luminosity ratio L_B/L_A.
- (e) In 3–5 sentences, summarize the full observable → model → inference chain used in this problem, including at least one assumption.

Problem 6 — Apparent vs. Absolute Magnitude Misconception

A student says: "Star X has $m = 4$ and star Y has $m = 9$, so star X is intrinsically brighter."

- (a) Explain why this conclusion is not always valid.
- (b) State what additional quantity you need to compare intrinsic brightness.
- (c) In one sentence, define apparent magnitude and absolute magnitude.

Problem 7 — Distance Modulus Practice

- (a) A star has $m = 11.2$ and $M = 1.2$. Compute its distance in parsecs.
- (b) A cluster is at $d = 250\,\mathrm{pc}$. A member star has apparent magnitude $m = 14.0$. Compute its absolute magnitude M.
- (c) Two stars differ by 5.0 magnitudes in apparent magnitude. By what factor do their observed fluxes differ?

Part B — HR Synthesis + Cumulative Midterm Bridge

Problem 8 — Main-Sequence Fitting by Magnitude Offset

A reference cluster at $d_{\mathrm{ref}} = 100\,\mathrm{pc}$ has a calibrated main sequence. A target cluster's main sequence is shifted fainter by $\Delta m = +3.0$ mag at the same colors.

- (a) Use distance-modulus reasoning to compute $d_{\mathrm{target}}/d_{\mathrm{ref}}$.
- (b) Compute d_{target} in parsecs.
- (c) If uncorrected extinction adds 0.5 mag of dimming, would your distance estimate from part (b) be too large or too small? Explain briefly.

Problem 9 — Bridge to Module 3: Mass, Luminosity, and Lifetime

Use the main-sequence scalings:

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{3.5}, \qquad \frac{t_{\rm MS}}{t_\odot} \approx \left(\frac{M}{M_\odot}\right)^{-2.5}$$

with $t_\odot = 10\,\mathrm{Gyr}$.

- (a) Estimate the main-sequence lifetimes of a $2.0\,M_\odot$ star and a $5.0\,M_\odot$ star.
- (b) A cluster's turnoff mass is about $2.0\,M_\odot$. Estimate the cluster age.
- (c) Why does this scaling imply that very massive stars are rare in old clusters?

Problem 10 — Cumulative Inference: One System, Multiple Models

You observe a binary system in a cluster and measure:

- Orbital period: $P = 4.0\,\mathrm{yr}$
- Semi-major axis: $a = 4.0\,\mathrm{AU}$
- Component velocity amplitudes: $K_1 = 25\,\mathrm{km/s}$, $K_2 = 20\,\mathrm{km/s}$
- Cluster distance modulus: $m - M = 10.0$
- Combined apparent magnitude of the binary: $m = 8.0$
- (a) Compute the total mass and the individual masses.
- (b) Compute the combined absolute magnitude of the binary.
- (c) In 3–6 sentences, describe what is measured directly versus inferred in this workflow.
- (d) Name one additional observation that would reduce uncertainty in the physical interpretation, and explain why.